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ORIGINAL DEMONSTRATIONS IN GEOMETRY.*

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It has often been observed that working original exercises in geometry constitutes one of the best means at our command of cultivating in pupils the power of clear, logical thinking. The question now raised is whether there is not some way of giving this educational discipline a deeper significance and of making its fruits available to practically all pupils.

It is also a common remark that in geometry the pupil has before him a set or kit of tools, viz., the triangle, line, point, etc., which he must learn to use; that in working an original exercise he must realize what tools he has before him and learn to select that tool or combination of tools best fitted for the work in hand. Perhaps something can be gained by carrying out this idea further than is customary, by studying the utilities and properties of each geometric implement, getting the most practical order in which to master these implements, and the most advantageous way of taking up each one.

The question at once arises, which tool occupies the primary place? Which is used most, is grasped most readily by the pupil, and logically occupies the most central position? An examination of the 165 text theorems in a standard Plane Geometry shows that the triangle is used 95 times as an aid in demonstrations, while the next most common tool, parallel lines, is employed only 40 times. Every attentive teacher must also have noticed that after a pupil has learned the use of the triangle he prefers to use this tool wherever possible. The triangle seems to mediate between the simpler elements, the point and line, on the one side, and the more complex, the polygon, the circle, etc., on the other.

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In like manner we find that far more applications are made of equal or congruent triangles than of the triangle in any other form. It seems natural then in teaching originals in geometry to let the pupil master the use of congruent triangles first. Let this be done by arranging the propositions of the text so as to use the triangle as early and freely as possible, and also by arranging original exercises in groups, putting first a group of exercises in proving triangles equal. If the pupil is weak and cannot use equal triangles in the seven or eight forms in which they appear, limit the subject matter at first so that he has to choose between only two or three of the tool forms of equal triangles.

After the pupil has acquired the power of proving triangles equal, let him take up a group of exercises in which he utilizes this power in proving a pair of lines equal by means of equal triangles. This constitutes both a review and an advance. Next give him a group of exercises in which he is to prove a pair of angles equal by use of equal triangles, thus putting the tool whose use has just been mastered to a new application.

After mastering the use of the first geometric instrument thoroughly in this threefold way, the pupil can proceed more rapidly. The use of parallel lines comes next, both naturally to the pupil and logically in relation to the subject, since two parallel lines and a transversal form but a special case of the triangle. After this the pupil can be made to feel that the locus, for some purposes, is a more efficient tool than the triangle. Thus ask him to prove that the diagonals of a rhombus are perpendicular to each other. He will naturally do so by the use of triangles. Then bring him to realize that the desired result can be obtained with half the labor, or less, by the use of the locus, and he will give this new tool a cordial welcome.

At this point it is usually well to introduce from algebra the use of a symbol for an unknown quantity and of the equation, and have the pupil realize that these tools learned in connection with another subject are useful in geometry also.

So far no use of auxiliary lines will have been made. But now that we take up their use, a group of exercises will be given, each of which calls for one or more such lines and the pupil will be kept at work with this tool till it is mastered thoroughly. Similarly, the mastery of various other geometric tools, as inscribed angles, similar triangles, analysis, symmetry, etc., may be acquired Groups of miscellaneous exercises are occasionally to be intro-

duced compelling pupils to use in combination all the instruments hitherto mastered.

In some cases it is worth while to bring out the toolage properties of a single theorem. Thus from the fact that two triangles are congruent if the three sides of one triangle are equal respectively to the three sides of the other, it follows that a triangular frame of iron or steel is rigid, even though hinged at its vertices. This is the unit of rigidity in girders, bridges and frameworks of all kinds and is of vast practical importance. The value and use of this unit tool can be brought out by problems like the following: Will a quadrilateral frame hinged at its vertices be rigid? Will it be rigid if two points, one in each of two adjacent sides, be joined by a bar, and if so, why?

Sometimes tool relations (often reciprocal) can be mastered in groups. Triangles are useful in proving the equality of lines and angles. Lines and angles are useful in proving the equality of triangles. Triangles are useful in proving lines proportional, and vice versa. Lines and points determine each other reciprocally; so of lines and planes, and points and planes. Some classes are much interested in making lists of the serviceable properties of each gometric tool, or combination of tools, and making comparisons of these lists.

Beside original exercises in direct demonstrations and constructions of which alone we have been speaking thus far, there are other original, or semi-original exercises to which the same method of treatment applies, the development of power to handle successive tools in these different lines being contributory to one common end.

Geometric drawing and observational geometry, even though treated in a very limited way, may be made to illustrate the tool functions of the point and line. Geometric drawing constantly illustrates the fact that two points determine an entire straight line, three points determine three lines, and the greater the number of points, the greater the possible efficiency of each point.

So also the drawing of lines and angles may be made to illustrate the utility meaning of the axioms, and show that the axioms are to be regarded not so much as fundamental equivalences as fundamental utilities.

Similarly the economic functions of straight lines may be brought out from another point of view by showing their utility in the mensuration of areas and volumes. To make two or three

linear measurements of a tank involves far less labor than to dip oil out of the tank and count the number of gallons dipped out. Hence questions like the following arise: The measurement of how many and what lines is adequate for the mensuration of the area of each plane figure, and for that of the surface and volume of each solid figure? Under what circumstances is it advantageous to use one linear measurement or one set of linear measurements rather than another? Thus, given that the mensuration of the area of the circle depends on the measurement of one of three lines, the radius, diameter or circumference, which of these three lines is it most convenient to measure in determining the area of the cross section of a pipe whose ends are inaccessible (as in the hot air pipe of a furnace)? In so simple a case even, it is with a distinct thrill of appreciation and sense of new power that the pupil realizes that it is best to obtain the area by measuring the circumference.

Other concrete applications give a new and deeper grasp of the toolage properties of lines in relation to other objects. For example, compare transportation by rail with that by water. A railroad train approximates the line in form and needs a comparatively large and scattered crew and much apparatus to manage it. A steamboat has three dimensions and is, so to speak, concentrated near a point, whence it can be managed by a small crew. This difference is the chief source of superior economy in transportation by water over that by rail, and it is a principle which has important applications in many other fields.

In like manner the study of numerical exercises in geometry may be facilitated and made more significant by converting it into the mastery of a succession of tools. We may separate the power to handle the formulas from the power to carry on difficult numerical computations and acquire each power separately. First have groups of exercises involving the numerical relations of various geometric objects in all their combinations and permutations, but in terms of small and exactly related numbers. Afterward have exercises adapted to develop the power of carrying on a long and complicated numerical computation. In this connection, but to a certain extent apart from it, groups of exercises adapted to give mastery of the most advantageous order of operations, of cancellation and other useful devices in computations, are to be worked.

A word may now be said about the application to actual classroom work of the above method of studying original exercises,

and then something as to results obtained. In Book I as soon as the pupil by study of the text propositions has become acquainted with what is meant by a demonstration and with the use of the triangle, the class may be put to work exclusively on easy originals in proving triangles equal and in proving lines and angles equal by means of equal triangles. From four to six exercises may be assigned at a lesson, and three or four more solved as sight work, either at the blackboard by the entire class under the supervision of the teacher, or on paper as test work. Thus, after the class is under way, practically an entire group of exercises is covered at a lesson. After the class has learned to use congruent triangles in original exercises, we return to the study of text theorems, but in connection with each daily lesson thereafter more or less original work is done in various ways, as by assignment as part of the regular lesson, or as sight work. At the end of Book I, more daily work in originals alone is taken up, and similarly throughout the subject.

By this means, every pupil who can learn any geometry learns to do originals, usually takes more interest in solving them than in studying the text propositions and succeeds better at it. In my own work no pupil is passed in Plane or Solid Geometry, or is certified as prepared to take a college entrance examination in one of these subjects, till he has passed a test in originals alone, as well as one in numerical exercises, and one in text propositions.

Beside these direct results, the method of treating the subject outlined above gives other results which are of interest. It seems to have a stimulating and vitalizing effect on the whole subject of Geometry. Many proofs which before seemed forced and arbitrary now appear direct and natural. Every proposition comes to have meaning and value. The method also seems to have an invigorating and vitalizing effect on the study of branches of mathematics taken up subsequently, as on Trigonometry and Higher Algebra.

Some help is given toward the solution of the problem of the relation of intuition to logic in the study of geometry. For my part, I do not see how we can dispense with some intuition, that is preliminary visualizing or sensualizing, in geometry, or in any subject for that matter. For instance, the idea even of a straight line or a circle is a highly complex one when considered apart from the intuition which we have of such an object. How

do we know by pure logic that it is possible for a line to exist, all points of which are equidistant from a given point? But if the pupil makes a start somewhere in geometry, as near first principles as possible, dealing with objects as he directly perceives them, for the sake of clearly realized advantages, he will be ready later to deepen and sharpen his logic for the sake of other plain advantages.

The history of geometry comes to have new interest. It becomes the study of a progressive series of new and improving tools. As a result of such study, the pupil sometimes imbibes some of the dialectic of history and is seized with the ambition to invent new tools for himself. The problem of combining individual instruction with class instruction is also partly solved. If originals be grouped in some such way as we have indicated, it is possible to assign as a lesson, say three easier originals for the whole class and two more difficult ones to be worked by the stronger pupils only. The whole class advances together, but its more gifted members gain a wider and deeper grasp of the subject than the rest of the class. The method is thus an important economy to the teacher who has large classes to conduct.

In like manner it prevents the somewhat narrowing effect which the study of geometry is apt to have. If the pupil forms the habit of acquiring the mastery of a succession of abstract tools whose values are consciously realized, he should be ready to welcome a new and better tool wherever found. Hence we arrive at a correlation of studies which is not superficial and concrete, and so full of miscellaneous detail that someone has termed it not correlation, but conglomeration; but at a unification by means of underlying utility principles. For instance, if the pupil in studying geometry acquires the habit of selecting the best linear measurements on which to make the mensuration of the circle, cylinder, etc., depend, when he comes to physics he will be ready to apply the same idea to the measurement of physical magnitudes. If he learns to realize the advantage of using auxiliary lines in the vivid field of geometry, he should be ready to use auxiliary objects freely in physics, chemistry, engineering, in ethics, sociology, and, in fact, find in such use a principle having a hundred applications for him every day.

Before I close allow me to say a word on one other aspect of the subject in hand. For the past generation, educational theory and practice have been mainly inspired and controlled by the

doctrine of evolution. The evolutionary philosophy of education has taken up and dominated other ideas, as those of Herbart and Froebel. We owe to this philosophy of education much of theoretic value and practical worth. But as a whole, its tendency is brutal and materialistic. In the nature of things it is but a preliminary step. For we no sooner announce the doctrine of the survival of the fittest, than the question arises, what is fitness? what is value? what is worth? Hence a new philosophy, termed Pragmatism, is springing up which is trying to answer these questions and give a broader and higher view of things, which shall include the doctrine of evolution as a mere detail. It certainly would seem to be a gain to analyze fitness (or worth) into the elements which compose it and then derive economical and efficient ways of mastering these elements. Perhaps also after we made this analysis we may be able to get some more fundamental and inclusive catagory than figures. Something worked out along this line must in the end profoundly influence educational theory and practice.

As a step in the new direction, let us make a distinction between concrete and abstract utilities. The use of similar triangles to determine the height of a steeple is an example of concrete utility, or of putting a geometric tool to a concrete use. The use of similar triangles to prove a set of lines proportional or a pair of polygons similar, is an example of abstract utility. Now if the pupil's mind be fed on the concrete applications of mathematics alone, or preferentially, a low grade and materialistic appetite is generated in him. His outlook and tastes are apt to be narrowed and he will be satisfied with nothing that does not have immediate material application. If, on the contrary, he gets an insight into abstract utilities, he finds that these are comprehensive and enlarging. They include practical applications as details, and suggest other particular concrete as well as other more general abstract utilities. Now the method of teaching originals presented above may be termed the method of abstract utilities, inasmuch as it puts abstract uses in the first place. Concrete utilities are brought in occasionally to sharpen and correct conceptions, and make interest more vivid; but abstract utilities are omnipresent and controlling, and fashion the development of the subject. When thus realized the method which we here advocate is seen to have new and wider values than those mentioned hitherto. The ends which

we aim at in the study of geometry are, first, practical results; second, general culture. The ancient Egyptians, in their use of geometry in land surveying, temple building, barn measuring, sought only the former. The Greeks aimed only at the latter. We desire both. In attaining the mastery of abstract utilities we, in a measure, attain both. The idea of abstract values mediates and harmonizes both kinds of value, and causes them to interact in multiplicative ways.

Hence it is suggested that it is a method which has a field of application wherever we find it desirable or necessary to combine technical and culture studies. In this age when new arts and sciences are raining down from the sky so fast that one is kept busy dodging them to save himself from destruction, if we can discover in each department a certain idealistic toolage, master this in one department and use it in others, it will be the source of much needed economy and uplifting power. It may be, therefore, that the method here suggested is a step toward meeting some of our educational needs in their larger aspect.

CULTURE COURSES IN THE SCIENCES.—GEOLOGY.

By William P. Holt,

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"Shall Geology be taught in the high school?" I think we shall all agree at the start that it depends largely on what kind of geology is intended. If it is to be a dry, technical course, attempting to make specialists out of high school pupils, or a course giving much attention to historical geology or paleontology, I believe that every live teacher of natural sciences would assert himself very emphatically in the negative. If it is the kind of geology than LeConte or Shaler would have given to high school juniors or seniors, fitting the subject to the age of the younger students, this is truly a very different proposition.

The courses these men offered to the thousands who elected their instruction were *culture*—not technical—*courses* of a high order. Like the work of Louis Agassiz their instruction will last through many generations yet to come. They taught a *great deal;* and, best of all, they inspired in their students that love for natural science which was only begun—not killed—by their instruction.

The kind of high school geology that we need to-day is the kind that will make our younger high school friends "more in sympathy with nature, more keen to analyze the problems presented to them daily—and to give them a broader and more human outlook" on their surroundings.

By statements made by Joseph LeConte, one of the *great* geological teachers of the century, I gather that these were his aims: (1) To interest the pupil, and at the same time convey real scientific knowledge; (2) To awaken the faculty, and cultivate the habit, of observation, by directing the pupil's attention to geological phenomena, and to geological agencies at work *now* on every side, and in the *most familiar things, especially in his local surroundings.* Referring to these ends LeConte remarked that it was by the first that he hoped to awaken a true scientific appetite; by the second to cultivate the habits necessary to satisfy that appetite.

It is this kind of geology that brings many earnest high school graduates back to tell their former teachers that they have never lost the fondness for the subject which they formed during their high school course—and that the physical world of nature has come to mean so much more to them. Such a course—a broad culture course if you please—was the *beginning not the end,* of *their* geological interest.

The teachers, however, who have the reward of getting such voluntary statements from their graduate friends are not those whose primary aim has been to cram their students for entrance examinations, or who have *attempted* to begin the making of specialists in the high school.

A word as to when such a course should be given. With all the work now required for graduation we can scarcely hope for more than a half year of elective geology in most high schools. This very plainly should not come before the junior or senior year. At that time, after the student has had an introduction to chemistry and physics, and other science work, the geology necessarily will mean so much more to him. Moreover the average high school student is so young that it is well to have the geology come as late in the course as possible—if he is to get the most out of it. Lastly, is it not well that at least a year should intervene between the physical geography and the elective geology?

Geology, as an elective in high school, is, of course, no experiment; and I have yet to learn of *one* case where such an elective geology course has not proven highly satisfactory where given by one deeply interested in the subject and able to fit the work to the students.

Now as to geology to be taught outside the geology courses proper.

In all of our high school texts in Physical Geography there is more or less of structural and dynamical geology worked in, although not, of course, under these names. A consideration of the dynamical, or earth modifying forces, must necessarily go hand in hand with a study of the present physiography to explain the latter. Would it not, moreover, be better if more exercises in simple structural geology be worked in—especially of a local nature—in place of many of the dry, uninteresting exercises that are sometimes imposed on freshmen in physical geography? I refer to such exercises as attempting to determine the latitude of the school building, to measure the number of miles between places on the map or globe, to determine the sun's altitude at different times of day, or to introduce problems in mechanics, having no practical application. One so-called "Hand-book and Manual of Physiography" dabbles in bits of Chemistry, and introduces the manufacture of oxygen, and other similar experiments having even less possible bearing on the subjects *supposed* to be taught.

Exercises of the former type kill the course for young students, and in the opinion of most successful teachers that I have met, have mighty little value for young freshmen; the latter, merely entertaining experiments, belittle the subject by making it a dumping ground for the odds and ends of all other sciences—regardless of whether they have any important bearing, or not. Do we not need in the first year of Physical Geography more of *earth science*, and less of the odds and ends of mechanics, astronomy, or plant and animal distribution? These subjects may all come in their appropriate places; but the successful teacher of physical geography is not called on to be a jack-of-all trades; he cannot afford to become such a dabbler.

Many of the more simple, practical parts of structural geology are entitled to a place in the first year's course in earth science—whether it goes under the name of physiography, physical geography, or earth science. It appears to me that there is a far

greater culture value in the study of many simple matters in local geology, in the first year of earth science, than there is in the study of the distribution and types of mountains in Africa or Asia.

How many pupils go through high school without being able to recognize with certainty the most common kinds of sedimentary rocks; or without knowing the kind of bed-rock underneath them; or the economic values of certain natural deposits or formations of their immediate regions? It certainly is far more difficult to properly interest pupils in far-away things that they have never seen—and many of which they never will see—than it is to interest them in their immediate surroundings.

A knowledge of the more common kinds of rocks that the student is likely to find either in the scattered surface boulders, or of the rock in situ; of the more common kinds of building stones of the locality; of the local ores received, if near a lake port; of local formations of especial interest, or deposits of economic value —these, together with a knowledge of the more common minerals, are some of the many things that may well be included in the first year of earth science, in connection with the physiography (proper) of the region.

If by a study of these and other matters in local geography and geology, together with carefully planned and well conducted field trips, we can interest our pupils more deeply in their surroundings we can then hope and strive to extend this interest to broader fields. It appears to me that there is little danger of over-estimating the importance of studying and teaching the local problems of greatest physiographic and geological interest—adapting the work, of course, to the age of those with whom we are working.

Can not we, as science teachers, co-operate in developing an even greater interest in local conditions? In Michigan, a carefully arranged collection of rocks and minerals, found in that state, has been prepared by the Michigan School of Mines at Houghton—and this collection, together with an accompanying manual, is supplied to a large number of the high schools of Michigan that have good claims upon same.

Such a collection for Ohio would be immensely helpful and instructive; and it might be arranged through our state geologist, or perchance the geological department of the Ohio State University. Each progressive, active teacher, who has worked over

his region carefully, might co-operate with the one having this matter in charge by furnishing samples of the rocks, minerals, natural resources, and other things of local interest; and might also furnish field photographs to illustrate points of interest in local geography and geology. This co-operation of the teachers interested in making high school earth science attractive, helpful, and stimulating, might bear rich results in this, as well as other matters, if properly backed up and supported. Are you ready to help?

Doubtless many teachers are familiar with the Washington School Collections of 40 minerals and 40 rocks, accompanied by a simple, but reliable, descriptive manual. The extremely low price of these individual sets (25 cts. each for the minerals, or the rocks) makes it possible for each pupil to have one or both of these sets for his own use. These are the collections that our former U. S. Commissioner of Education, W. T. Harris, said should be in every school in the United States—that he had never seen anything better for acquainting the school youth with the more important rocks and minerals and giving them some idea of the significance of these in the earth's history, and in the industries of the people.

Probably most teachers would only care to cover a part of these during the freshman year of earth science but both collections could be carefully gone over later, in the senior or junior elective course in geology, as a part of the laboratory work; and these would be a great help in connection with the collections of local rocks and minerals that each student should be required to make in the field.

In addition to the study of local conditions, and a somewhat more general—though brief—study of the rocks and minerals of a larger area, is it not well to introduce some of the more important *ideas* of modern geology into the first year of earth science? *How* "the waste of the old land becomes the material for the new," and the relative age and arrangement of the strata, together with the ideas of continuous, though imperceptibly slow, changes which are taking place everywhere as a resultant of constructive and destructive forces—and the *enormous* periods of time involved—these, and many other truths of modern geology, have a far greater interest and culture value, to even young students, if properly presented, that many of the dry "paper

exercises" in physical geography sometimes inflicted on young, first-year students.

Like other great subjects such truths must be approached cautiously—a step at a time, and *slowly* at first—but if such geological truths are carefully and forcibly presented they will not only deeply interest the average high school pupil but will afford a definite culture value, and a lasting benefit.

In conclusion we finally urge that:

1. A course in geology, elective for juniors and seniors, is *highly* desirable as a culture course, to bring thoughtful students more closely in touch with their surroundings, and to make them observe more closely, and think more carefully concerning the great physical world around them. That such a course, if taught by a live teacher, who loves the subject, should be the beginning —and not the end—of the student's interest in such matters.

2. That Physical Geography is so inseparably connected with dynamical and structural geology that many parts of both these need to be introduced into the first year of earth science. That, if properly presented and adapted to the ability of the students, these introductions from geology may do much toward developing a deep interest in both Physical Geography and Geology. And it is further contended that there is more practical and cultural value in the study of same than in many of the dry, uninteresting "paper exercises" of the indoor laboratory.

3. That teachers of a given region, or of the entire state, should *co-operate* in developing a greater interest in *local* earth science. That a state collection of minerals, rocks, natural resources, together with a collection of local field photographs from active field workers is highly desirable in the teaching of high school geology and earth science.

A BIG SPRING.

No state in the Union has larger or more numerous springs than Florida. Many of them form good sized streams from the start and some of them are navigable. The largest spring in the state, and one of the largest and probably best known in the United States, is Silver Spring, which is located 6 miles east of Ocala. This spring forms the source of the Oklawaha River, a tributary of the St. Johns, and steamboats traversing the river enter the spring basin, which has an area of several acres. The water is from 25 to 30 feet deep and is wonderfully clear, appearing absolutely colorless.—*U. S. Geol. Sur. Press Bul. 292.*

PHYSICS IN THE SECONDARY SCHOOL.

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The experience of many schools has demonstrated the fact that it is impractical to attempt to prepare boys for the College Entrance Examination in Physics in one year. Even to do the work in a cramming sort of fashion with certainty of good results takes more time than ought to be given to a single subject. But, what is of still greater importance, to mature the new conceptions involved, requires in the mind of the average boy of sixteen to eighteen more than a single year. Mental development is a growth, and cannot be forced without bad effects on mind or health. It may not yet be scientifically demonstrated, but certainly it has proved the case in numberless instances, that for growing boys, too highly specialized mental exercise, as well as too heavy physical exercise, intereferes with a normal, well-rounded development.

From the psychological standpoint, the order of presentation of new conceptions is also of importance, and in a one-year course it is not possible adequately to take this into consideration. It seems now to be a generally accepted conclusion that an unconscious, but very real, organizing and harmonizing process goes on in the brain, which tends to establish orderly associations between all experiences, educational and otherwise. In some branches, such as history, this idea has been put into practice by giving, in the lower grades, first mythology and legend, then historic or biographical stories; and finally, after a rest period of one or two years, full-fledged college entrance courses in Greek and Roman, and in English and American history, with note-books, outlines, and outside reading. In English, only the simplest ideas of the structure of a sentence can be taught to advantage in the early years; later all the parts of speech and their use are taken up, together with the analysis of sentences, and so forth; and finally, after a period of comparative rest, during which there is only practice in applying the principles already learned, rhetoric is studied in connection with the work of preparation for college entrance examinations.

In science, also, the same attention should be given to this fundamental principle. My own teaching experience of ten

years has shown me that it takes time, repetition, and many concrete examples to build up in the mind anything approaching to a sound conception of a law or a principle. It must be remembered, too, that while activity is an essential characteristic of youth, variety is the soul of activity. Therefore, remaining too long on one phase of a subject should be avoided. Yet, in going from one phase to another, it is of the utmost importance, especially on the first presentation, that the two phases be naturally and obviously related, or else be so presented that the previously acquired general experience of the boy is a sufficient foundation for the new conception.

If the data be presented by laboratory experiments or demonstrations, we must insist that time is required for absorption, assimilation, and incorporation into the mind, otherwise confusion is sure to result; and confused conceptions and blurred reasoning processes are the bane of our present varied and spread-thin education, and an almost certain bar to future success.

Prof. Woodhull has said that in the Horace Mann School they had found it necessary, in preparing boys for the college entrance examinations in physics, to have a separate division with extra work added after the completion of the high school course. They give more periods a week to physics than most schools do, if I am not much mistaken. (Three double periods, *i. e.* six ordinary periods per week.)

Granting then that more than one year is necessary to properly prepare the average boy in physics, of what shall the work of the preliminary year consist? Evidently, some practice in making accurate measurements, and a working familiarity with the metric system is necessary. The English have "Physiography," an outgrowth, I suspect, of the natural philosophy of our grandfathers' school days. A good deal might be said in favor of this plan; for, on the one hand, it is the logical next step after the ordinary physical geography course, as it carries this on in a somewhat more scientific manner, and into a broader field. This course discusses: 1st, The earth from the astronomical point of view; next, the earth from the geological point of view; and, finally, some of the fundamentals of physics and chemistry. The purpose of such a course is the development of interest in the processes of nature going on around the boy, and of the feeling that science is not, as far as he is concerned, something

remote, abstract, useless, and impractical, but rather that it is very vital and definite. When he has in this way surveyed the field occupied by each of the sciences named, he should have some inkling of the care and accuracy they require, of the methods they employ, and of the principles they have evolved. He should have acquired some idea of the relatedness of things, and should not be left without a notion of what it is all about, and of what it all amounts to; as is sure to be the case if he plunges into the forty experiments in the usual way; where he is dragged along and left dangling, only to be seized and dragged again to some dizzy height or stupid detail, being left finally without any adequate conception of the connection between the two. The abstract principle is too thin air for him, and the detail is too prosaic, in the usual disconnected form of presentation.

To overcome these difficulties as far as possible, the following six year course has been adopted in the Morristown School with good results: 1st year, geography, drawing and manual training; 2d year, commercial geography, design and manual training; 3d year, physical geography; 4th year, elementary physics, with drill in the rudiments of chemistry; 5th year, chemistry (if both physics and chemistry be taken); 6th year, physics (the forty experiments).

As to the correlation of studies, some mutual benefit has been derived by co-operation between the English and geography teachers during the first and second years; and the drawing teacher renders much service by instruction and practice in map drawing. These aids also apply to the physical geography of the third year. The manual training course of two years is also a great help to the physics which comes later; and those boys who take mechanical drawing and special manual training work, of course, find these of great assistance in their laboratory work. The mutual assistance between the mathematics and physics courses is almost too obvious to mention.

PHYSIOLOGY, HOW AND HOW MUCH?

CLIFFORD CROSBY,

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What is the matter with physiology? Unless it can be granted that this subject is not a success at present in the high school, my paper has no basis for a hearing, because I wish to advocate that physiology as a separate subject in the high school curriculum be done away with. Let us inquire then what the facts are which warrant any valued objection to the present status of high school physiology. In the first place a number of the principals of the larger high schools of Iowa have stated that physiology is not a success in their schools. Mr. Smart of Davenport says: "As physiology is taught in most high schools, it is a farce as far as science is concerned." Mr. A. C. Butler, a prominent school man, known to some of you as the author of "Persimmons," conducted a test in his school to ascertain how students viewed the subjects in the course on the basis of like and dislike. He discovered that out of one hundred students in the physiology classes, seventy of them found the subject to be the most disagreeable one they had, while only three of the entire number named it as their most interesting subject. Further, in a list of questions which I recently sent to seventy of the high school principals of Iowa I asked the question, 'Do your pupils like physiology as well as they like other laboratory sciences?" Of those who answered the question, only twenty-seven per cent replied "yes"; seventy-three per cent said "no," while others, perhaps out of courtesy to the teacher in charge, expressed various kinds of dissatisfaction with the subject without stating plainly that it was disliked by the students. Now, I have faith enough in students to believe that a general dislike for a subject means that for some cause the student is unable to find a valued content in the subject, and if this be true, we are committing an error by teaching physiology in the way it is now taught. I am not speaking thus without personal experience in the matter, for I remember this subject from both the standpoint of the pupil and the teacher.

What, then, is the difficulty in getting the desired results from this subject? Surely its content is of sufficient importance! To answer the question, reference will again be made to the data recently obtained from the Iowa high school principals.

Is physiology taught as a separate subject?
What science work is given the first year?
What science work is given the second year?
What science work is given the third year?
What science work is given the fourth year?

From this material the following facts were derived:

1. Schools not offering the subject, 14 per cent.
2. Schools offering physiology after botany, 37 per cent.
3. Schools offering physiology after zoölogy, 19 per cent.
4. Schools offering physiology in combination with zoölogy, two cases.
5. Schools offering physiology before either botany or zoölogy, 52 per cent.
6. Physiology is taught during the first year in 50 per cent of the cases reported.

An examination of these facts reveals the following faults. Not only is the physiology of man, a vertebrate animal, attempted in an entirely unconnected way so far as concerns the other related sciences, botany and zoölogy, but it is usually taught to students who have, as yet, no knowledge of the general physiology or morphology of lower forms. It has doubtless occurred to all teachers of the subject under consideration that the dividing line between zoölogy and human physiology was a superficial one being based mainly on the species of animal under consideration and not upon any distinguishing physiological feature. It seems strange to attempt a detailed study covering an extended length of time of the most complex of animals without first getting a general view of the animal kingdom in all its phases. What Professor Hall says is explicitly true, namely, "Physiology should be the biology of man rather than simply human physiology."* When such a view of the subject is accepted, it becomes impossible to ignore utterly the natural relationships of human physiology to other biological sciences, as has been done apparently in the past. The fact is, that too little attention has been paid the subject, so that even in the larger high schools it occupies often the position of "time filler" for the teacher who happens to have sufficient time for the work. The matter of the teacher's preparation is thus ignored.

To the present method of handling the subject there is a number of objections. First, physiology is not a laboratory subject

*School Science, April, 1901.

as it is usually taught, for in most cases it consists merely in the assignment and recitation of printed matter from a text-book. True, a few physical and chemical experiments may be performed and some demonstrations on lower vertebrates may be shown, but aside from a small number of experiments on digestion, little laboratory work except that drawn from other sciences can be done. In a carefully worked out paper on "Physiology in the Horace Mann High School*" of New York, there is included a detailed list of experiments and dissections to be performed. Their classification is interesting. The summary is as follows:

Experiments of a chemical nature, 18.

Dissections, demonstrations, etc., dealing with lower forms, 33.

Exercises based on the human body, 4.

Evidently human physiology alone cannot furnish suitable laboratory work. This is necessarily the case on account of the unusual difficulty of the subject. The fundamental principles of physiology, such as respiration, nutrition, irritability, etc., which are easily demonstrated with botanical material or lower zoölogical forms, are entirely beyond the reach of high school laboratory studies in humans. Also the anatomical facts are nearly all beyond the reach of laboratory demonstration, with the exception of those concerning the skeleton and the external features. If these statements are true, then human physiology does not stand on the same plane with our other laboratory sciences, in that it fails to give the inductive scientific training, such as is furnished by physics, chemistry, and biology. For example, compare the training involved in working out experimentally Boyle's Law, or a little problem in botany, like the effect of water on some particular kind of root, with that obtained in memorizing the average paragraph in a physiology text—say the one on "the effect of alcohol on the liver." If it is the purpose then to give four years' consecutive training in scientific method in the high school, it is evident that present day physiology cannot be included in such a course.

Although we ordinarily speak of human physiology as "physiology," it is evident that it must include more than physiology, strictly speaking. Anatomy is an important part of every text on the subject, about one-eighth of each text examined being devoted to that subject, the remaining seven-eighths treating of

*Journal of Applied Microscopy, p. 320, Peabody.

the physiology of the organs and the proper care of the same. There is a distinct necessity for this anatomical part of the text, namely, that pupils have never seen any of the structures spoken of or even the corresponding organs of lower vertebrates. Now, the above proportion seems to be about the right one, as the physiology of any organ in plant or animal is more interesting to all classes of students than the anatomical facts concerning it. Besides it is the more important phase of life, and utilitarian ideas are more numerous in this division of the work than elsewhere. However, this in no way lessens the importance of a certain amount of anatomical work, for upon this basis all physiology work rests. In other words, the organ and its function must be studied together. What sort of a basis in anatomical knowledge, then, may the student be supposed to have acquired from a twenty days' study of text anatomy? Practically none. Descriptive matter conveys but few pictures to the student's mind, when a general notion of the organ defined is lacking. If a student has a general notion of a lung, based on a few experiences with one, it would be possible to add sufficient detail to make a fairly clear picture, but with this lacking for a foundation, it seems impossible. Would it not be better to obtain from the lower forms some general ideals of the chief structures, which are homologous to those of the human body, before attempting to gain much from a text-book description of the same. Zoölogy will do this preparatory work on the lower forms and so afford the foundation necessary for successful comparisons with the human body, thus making possible a clear understanding of the necessary anatomical facts.

The essential principles of physiology, those which the student should carry away, are comparatively few in number and may be stated in a few pages, if not in a few lines, but the comprehension of these ideas is a rather difficult matter for the high school student. This can be accomplished by holding his attention on the ideas for a considerable length of time and approaching each one from a number of viewpoints. A mere memorizing of words will never suffice, as is shown by the following quotations from physiology examination papers. My authority is Miss Le Row.*

(a) A young person who goes to dances, parties, etc., generally has some brain trouble, like St. Vitus' Dance.

*"English as She is Taught." LeRow.

(b) Soap is important in carrying off the fat of the body.

(c) In a diet of twenty-four hours a man should eat some of all the nutritious articles.

(d) Respiration is the exchange of carbolic acid for oxygen.

(e) The covering of the brain are the Diameter, Perimeter, and Trachæ.

Continued laboratory work upon several forms, during which time the ideas are approached again and again from new viewpoints is the necessity, as this directs thought and energy to the required end continuously. How is this important end reached by our present methods? A few printed paragraphs state the essential truths, while the great mass of the book is devoted to other facts, which so far as is evident to the student, are of equal importance. As Herbert Spencer says, "Success in the world depends on energy rather than on information, and a policy, which in cramming, undermines energy, is self-defeating." The conditions described above are, therefore, fatal to desired results.

Of course to the above objections it may be stated that a course in zoölogy, preceding the human physiology, will give sufficient familiarity with the anatomical features to make the structures described in the text realistic, and also that these facts will become the nucleus about which physiological ideas may be hung. This is only true in part. A year or even a half year in zoölogy, if it includes three hours per week laboratory work, is sure to furnish some clear and lasting conceptions of the various systems and organs of other forms, which will serve as an invaluable basis for any text-book work along related lines, but it must not be forgotten that the students carry away a comparatively small amount of any subject, and then only the broadest generalizations. It cannot be expected, therefore, that such dry, unrelated things as anatomical detail will be remembered after a lapse of a few months. This vagueness is not compatible with the formation of clear pictures of human structures, and constitutes a strong objection to the separation of zoölogy and human physiology. The time for the consideration of the facts of human anatomy is when the homologous organs of the lower forms are clearly in mind, the exact detail of each not having been forgotten. Suppose that the student has just completed drawings of dissections of the nervous system of the cray-fish and frog, how easily will the ideas of the human nervous system lend themselves to study, when thus backed and emphasized by

vivid detail of similar structures! The involuntary comparison resulting will give not only clear ideas, but will add greatly to the interest in the subject.

My final objection to our present method of teaching human physiology is that the method of presentation is the same as that in the grades, whereas an entirely new method of approach is desirable. As far as my knowledge of the matter goes teachers are doing the physiology work in the grades in about the same way that it is done in the high school. The same threadbare experiments are performed, and the same demonstrations are made, the point of view remaining the same in all grades. Owing to the fact that this process has been going on through several grades the pupils as a rule are tired of the subject and have little respect for what they have learned. Even if the subject does contain new ideas, there is a distinct loss in beginning the subject with the drawback which previously formed opinion may have brought about. It seems then, that to approach the subject under the name of zoölogy, with the scissors, needle, and microscope well in advance, is not only the scientific method of approach, but also attracts and interests students as only the development of a new idea can.

This leads me to a more exact statement of what I consider to be the best method of giving high school pupils the main facts of physiology. It is this: Give a year's course in zoölogy, devoting a good share of the time to general physiology,* and while these facts of anatomy and principles of physiology are clearly in mind, make their application to human anatomy and human physiology. This plan seems, on the whole, to be a good one for several reasons.

In the first place, human physiology, on the basis of the importance of the subject-matter, is superior to zoölogy, hence the effect of the training and knowledge received in the latter should be used as the foundation for the important facts of human physiology. The relation between the two is close, indeed, because all of the principles of general physiology apply in the particular case of man, yet in our teaching of the subject we have not recognized this fact sufficiently. On the contrary, the detailed study of the most complex of animals, man, is either placed first, that is before any general view of biology has been obtained, or at best a line of separation has been drawn between

*Riddle, School Science, March and April, 1906.

it and zoölogy where none exists. Man seems to have been looked upon as a special organism, but this is certainly not the case; he is simply an animal and the physiological unity which exists among other animals is unbroken when man is included in the list, in spite of the customary treatment under separate covers. There can be nothing more natural, then, while physiological principles are under discussion, than simply to apply them to man.

Further, these essential processes of life are more easily understood in simple animals which can be observed in their entirety than in complex animals like man. Take a bird for example and try to study its nutrition; the alimentary tract may be traced and the disappearance of the liquid portions of the food noted, even a microscope will disclose little more. Or, try to study respiration. The respiratory tract may be traced to the lungs, but no explanation of respiration will be found there. In fact such a study will lead to the conclusion that digestion and absorption are the essential processes of nutrition, and that inhaling and exhaling constitute the fundamental processes of respiration. No. If nutrition is to be taught, watch a stranded paramoecium engulf carmine grains and then see the circulating protoplasm of the cell; or put a hydra into water from which all the air has been expelled and then cell respiration comes out of the realm of hazy facts to the stage of apprehension. The unicellular, or few-celled forms have little detail in which to hide the vital processes, or detract attention from the fundamental notions, yet in the life of each of them are operating all of the vital phenomena which are displayed in the larger animals. The student, therefore, who makes a study of such forms, is well prepared to understand the nutrition and respiration of the cells of a tissue. The closer this association is made, the more sure of results we become.

Then, the systems of man cannot be appreciated till the necessity for them is understood. In many single-celled forms the circulation of food in the organism can be seen. Advance a step, and the student finds the cells of the hydra feeding in an amoeboid way from a common body cavity, but when the larger animals are reached, the increased size of the organism and hence the remoteness of many cells from food, make the difficulty of such primitive food supply evident. Thus the purpose and necessity of the systems of the human body are understood

from both the evolutionary and physiological standpoints. Such a presentation of the various systems in the closely related way proposed is satisfactory from both the human physiology and the zoölogical points of view, for while such ideas are ordinary enough in any zoölogy course, the human physiology gains exceedingly in that the "history of the system" and work to be accomplished by it are understood.

From what has been said, it is clear that the ruling ideas in the course are physiological ones, and there is no doubt in my mind but that this is a satisfactory basis for unity. At one time I planned and partly executed a course in which the unity of the individual animals was subordinate to the study of some particular system. Thus, the circulatory, reproductive, and nervous systems were studied separately, as units, in the various types, a fresh start in the list of types being made when each new subject was taken up. This rather extreme plan was far from being entirely unsatisfactory, because vital processes instead of being subordinated, or lost sight of in a mass of details, were emphasized. From this I argue that a course of zoölogy from the lower animals through man, unified by the ideas of physiology which are common to all, cannot fail to be satisfactory.

It was noticeable in the replies to the inquiries above mentioned concerning the status of physiology in the Iowa high schools, that a considerable number of principals took occasion to state that their physiology was taught practically without any laboratory work. They laid the ordinary dislike of physiology to this cause and they were partly correct. Now if the suggestion which I make is carried out, this difficulty which exists of obtaining suitable laboratory work for human physiology will be obviated and the subject will come at once upon a laboratory basis. In this way human physiology may acquire a right to be considered a science capable of giving the scientific training which ought to be expected of it. Those of us who have taught subjects like physiology and physiography from a book, by assigning a certain number of pages each day, will welcome any plan which will relieve the dead monotony of such a course. Nor will our relief be any greater than that of the pupils. A thorough course in laboratory work, such as easily follows the introduction of zoölogy into the course with human physiology, is sure to result in an increase of interest in the subject.

The proposition of teaching physiology as a subject unrelated to zoölogy does not seem natural, in view of the fact that the closest correlation exists between the function of an organ and its structure, zoölogy being presumed to furnish knowledge of those structures. An examination of the attitude of the universities in this matter shows that the physiology courses in botany or zoölogy invariably follow rather complete work in anatomy, or in somewhat elementary work, the fundamentals of structure and physiology are given in the same course. The book method of teaching the subject necessarily separates these two, some texts actually placing the anatomical facts in separate chapters. But even if the physiology and anatomy are closely associated in the book, there never can be the close mental association which comes from having seen the organs in action, or from having considered their function while making a dissection.

The complete disuse of the ordinary physiology text, which is brought about by the union of the subject with zoölogy, results in giving the teacher greater liberty in handling the subject than any other method allows. There are a number of good effects resulting from this: First, some subjects, such as respiration, nutrition, and some of the special senses, are inadequately treated by the texts generally used, and this fault can be rectified. On the other hand, many details not essential to the subject, and which are remembered only long enough to be recited, may then be omitted without occasioning the detriment to the student which is brought about by omitting considerable quantities of any text. The "physiological effect of alcohol" may be cited as a prominent example of subjects which are best omitted from high school physiology. There is a good deal of this material in all our texts, and, so far as I am aware, the same objection may be made to all of it regardless of the author of the text. The method used in presenting this material has been to seize upon one principle and apply it to each organ and tissue of the body. The result of this is a degenerating repetition. The subject will not be susceptible, of course, to laboratory teaching, but it can be dealt with under the proposed plan, briefly and emphatically in any way the teacher sees fit, and it will probably be as effective as wearying repetitions in texts.

If human physiology and zoölogy be taught as a unit, there will be mutual gains resulting to each. Undoubtedly the high-

est value of zoölogy lies in the scientific training it gives, and this, as an object, while worthy is not tangible or concrete enough as an argument to the pupil himself, and, laying aside the bearing of zoölogy upon human physiology, there remains only "Economics of Animals" as a "practical"* argument for the subject, and this last is a weak one. Regardless as to whether human physiology is taught with zoölogy or not, the zoölogy will, to a varying extent, enrich the student's conceptions about his own body, but this effect will be greatly increased if an organic union of the subjects be made. Hence, since everyone recognizes the value of knowing something about the workings of one's own body, a sufficient utilitarian purpose will have been furnished the zoölogy, and at the same time human physiology, which by itself gives little scientific training, will acquire this from the more systematic laboratory work of zoölogy. Thus each subject is benefited.

Again, the time limits of the course preclude the teaching of both zoölogy and physiology separately. Allowing necessary time for physics, chemistry, physiography, and botany, there certainly cannot be required in addition time for a course in zoölogy followed by one in physiology, so that even though a zoölogy course, preliminary to physiology, might be ever so desirable, yet other necessities of the course prevent any such arrangement. The case is somewhat different in the larger high schools, where only a certain amount of science is required. There it is quite possible to offer zoölogy and physiology courses separately, as electives, in opposition to other science, electives, but this involves making a few years' zoölogy a requirement for those who would elect human physiology. At least such will be the case if the best results are to be obtained from the physiology. Now, this is too great an outla, of time for those who desire merely to get the main principles of physiology. It seems, then, that the union of zoölogy and human physiology is almost a necessity from the standpoint of the economy of time alone. Some such a union has already been effected in New York City, where the subjects under discussion and also botany have been united into a single year's work. I see no objection to this, if an additional year of elective botany is furnished, for I am well aware that botany, above all biological subjects, furnishes good

*Page 247 Lord & Bigelow, Teaching of Biology.

material for the demonstration of general physiological principles.

Finally, it can be shown that the subject of zoölogy is so closely related to human physiology that the majority of topics usually discussed in physiology texts, are part of the ordinary zoölogy work. I have here a list of topics taken from a chapter in an ordinary text:

Adenoid vegetations.
Respiration.
Respiratory organs.
Amount of air in each breath.
Modifications of breathing sounds.
Breathing sounds.
Effect of tight lacing.
Nose.
Mouth breathing.
Abdominal and thoracic breathing.
Respiratory center.
Inspiration and expiration.
Amount of expansion.
Rate of breathing.
Artificial breathing.
Oxidation.
Pharynx.
Larynx.
Trachea.
Bronchi.
Cilia.
Lungs.
Chest.
Action of cilia.

The following topics should be omitted as not being vital to the subject:

Topics like "adenoid vegetations" mean absolutely nothing to the student unless he has some special experience, and yet such topics engage as much of the student's time as the important ones. A second class of topics are laboratory topics and would not have been taken up in a zoölogy course if physiology were not included with it, but it is to be noted that they are of small consequence so far as the time spent in their discussion is concerned. Related to these last topics are certain "lecture topics," which must be added to any vertebrate zoölogy in which the human receives consideration. With few exceptions they are of minor importance, and the presentation of them all will take but little time. There now remain others constituting a majority, and all of them naturally arising in any course of zoölogy which is at least one year in length. In fact, their intelligetn consideration in human physiology involves laboratory work which is duplicated in zoölogy. Authors of physiologies have recognized this by indicating that such work might be done. Nor is this chapter

other than a typical one, for almost any one in the book will furnish these conclusions if similarly investigated.

The following summary may be made based on the above statements:

1. Human physiology is unsatisfactory to pupil and teacher as it is now taught.
2. The continuity of scientific training is broken by physiology.
3. The vital processes do not receive emphasis enough under the present system.
4. There are a number of pedagogical objections to physiology when taught "separate" from zoölogy.
5. Human physiology treats of the most specialized form of life and therefore ought not to be taught without regard to other biological sciences.
6. Physiology ideas form a suitable basis for unity in the combined course suggested.
7. Time is saved by the union of physiology and zoölogy.
8. The ideas of physiology and zoölogy are largely identical.

I conclude therefore that human physiology as a separate subject ought to be discontinued and that its ideas should be presented with the course in zoölogy.

DIAMONDS IN PLACE IN THE UNITED STATES.

Although diamonds have been found in at least thirty places in the United States, the only locality where they occur in place has recently been discovered and has been investigated by Mr. George F. Kunz, the gem expert, and Dr. H. S. Washington, petrographer. They occur in an igneous rock, similar to that of the South African mines, which forms a small stock near Murfreesboro, Pike County, Ark. The first two stones were found August 1, 1906, and since then many of them have been picked up, the total number found at last report being 130. Many are white and of good water, others are yellow, and some are of brown bort. The two largest stones weigh 6½ carats, one being exceedingly fine white and the other brown. They are found on the surface as well as within the greenish, friable, decomposed peridotite, a rock somewhat like the famous "blue ground" of Kimberly. The property is being actively prospected and developed.—*U. S. Geol. Survey, Press Bul. 292.*

LABORATORY NOTEBOOKS IN BIOLOGY.

By WORRALLO WHITNEY,

South Chicago High School, Chicago.

A laboratory is understood by all, I think, to mean a place where things are studied at first hand. But as to the manner in which the laboratory shall be conducted biologists do not agree. I am leaving out of account the use of the laboratory for investigation and speaking only of its use for students. Some biologists seek merely to have their pupils confirm what has been found out by others. Scientific training is incidental to this course. Other biologists regard the gaining of information as of secondary importance, and the training given by the laboratory as of first rank. This training teaches the pupil how to draw inferences or reason from observed facts and how to observe natural phenomena. In this case, then, the laboratory will not only give a body of facts, information to the pupil, but it does much more. It trains the pupil in scientific synthesis—the correct habit and method of drawing conclusions from known data. The text-book is an adjunct to the laboratory. The laboratory method is an attempt to follow the order of discovery so far as practicable within the limitations under which the teacher is placed with respect to size of classes and routine of school management.

The notebook has always been considered a very important companion of the laboratory student. It is the record of the observations and conclusions or inductions of the student. It is more than that. The notebook may properly be an incentive to better work for it will tend, when used rightly, to prevent careless work. When a record is to be made in the notebook, the observations will be made more carefully than when noted mentally only. The drawings, too, tend to beget more careful work on the part of the student and in this we see the reason for the high esteem the notebook holds. It is both the history and the incentive of the careful student. It is the pulsebeat of the laboratory, indicating as it surely does, what is done in the laboratory. The careless and careful students are accurately reflected by their notebooks and likewise the careful teacher may be distinguished by the notebooks turned out by his pupils.

It will contain, certainly, a record of the pupil's work. This goes without saying—so we are really bound to inquire as to what ought to go into its making. If we are following out the

ideals of those who would make the laboratory train as well as inform, the notebook will contain a record of studies of animals or plants—of actual first hand studies. It will not contain many copies of pictures from charts, lantern slides or books, for this would be information work pure and simple, and is only justified when it is necessary to fill in missing links, representing work that cannot be done in the laboratory. At most, copied work whether from drawings or from descriptions in books should form but a small percentage of the entire work. There can be no disciplinary value in it worth comparing with that obtained from actual study of the object and it is not worth while to spend much time copying such drawings just for the sake of the information. The teacher would do better either to choose a different line of work or to copy such drawings or information as he wishes himself, giving it to the pupil by means of the mimeograph.

The record contained in the notebook, then, will largely be of first hand observations. It will consist of sketches and notes. I should say practically everything studied by the pupil should be sketched. I think sketches far more valuable than notes both in disciplinary value in compelling the student to observe carefully and in faithfully recording what the student has actually seen. He may make a shrewd guess at the correct answer for his written notes, but he cannot guess where a line should be drawn in his sketch of an object; the line must go where the observer sees it or not at all. Therefore the sketches represent more faithfully the work of the pupil than the notes and are correspondingly more important.

The notes should form a setting for the sketches and should be a record of things observed. There should be as little copying of directions and explanations as possible. These are not notes, properly speaking and should not pass for such.

The manner in which the notes and sketches are prepared is very important. Our pupils are getting their first lessons in note making, hence we are responsible for the product. If the drawing made by the pupil is incorrect, it is worse than useless for it will tell an untruth to the pupil every time he sees it. Likewise incorrect notes do the same thing, but this fault does not stand out so plainly as in the case of the drawing. It is, then of the utmost importance that the sketches and notes should be carefully made. Careless, slipshod work tells nothing or little where it might tell much.

It follows that the teacher should not let any work pass which is incorrect and does not represent the very best work that the pupils can do. Of course the abilities of pupils vary greatly and likewise the quality of the work, but the teacher must hold each to his best. The poorest work can be approximately correct, so that it will not actually misrepresent the facts, though it may not be good work. Since the sketches are the most important and the notes are likely to be correct when the sketches are correct, the emphasis should be placed on this branch of the work. The teacher can very quickly inspect a drawing and they thus form the most convenient test.

It has been my experience that the drawings must be examined while they are being made so that mistakes may be corrected while the material is still before the pupil and the subject fresh in his mind. I have never found any other method effective. Merely indicating that a mistake has been made after the work has been handed in does not answer, for the pupil will usually guess at the correction needed or borrow from a classmate. Of late I have been using a rubber stamp and allow no drawing to be considered complete until I have stamped the plate "accepted." This does not conduce to absolute order and quiet, for each pupil must report with his drawing, but it is the only satisfactory method I have ever found to hold an effectual check upon careless work.

If we are to insist upon careful, accurate work in the preparation of notebooks, the notes and drawings should be made in ink. The habit of hasty scratch methods with the pencil is too strongly fixed in the pupil to be overcome. This year for the first time I have required all notes to be taken in ink in the laboratory and I have found the gain very great over any other method I ever tried. We get from our pupils just what we expect from them and insist upon. If we tell them to take their notes in ink and make no mistakes, they will be correspondingly careful. If we allow them to use a pencil, we may expect a good many mistakes. It pays to ask for the best right from the start.

We should require that all notes be written in good English sentences, which will make sense when read in the absence of the question which the pupil is answering.

THE SIPHON—TWO NEW LECTURE DEMONSTRATIONS.

By Will C. Baker,

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The following notes are communicated to School Science and Mathematics in the belief that they are novel. That the experiments effectively direct the attention of beginners to fundamental points is the experience of the writer.

I. Theory of the Siphon—a lantern demonstration.

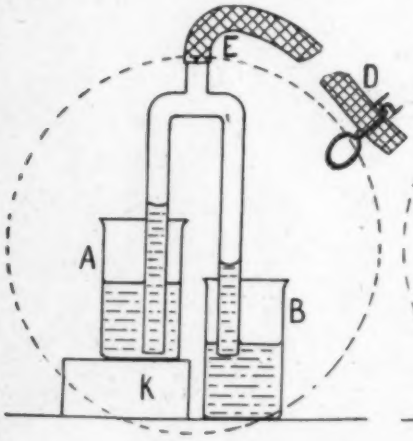


Fig. 1.

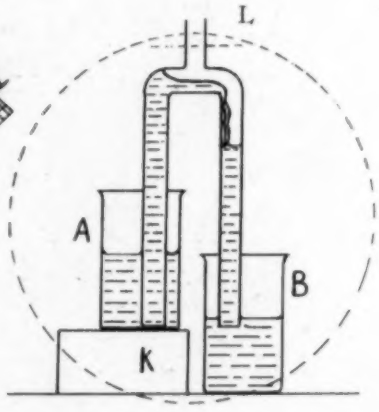


Fig. 2.

The experimental arrangement is shown in figure 1 where the dotted circle indicates the condensing lens of the lantern. A and B are two small beakers at different levels, containing water. Into these is placed an inverted U tube having a branch at the top as shown. A piece of rubber tubing E a couple of feet long (with a spring clip at D) enables the operator to draw the water into the tubes as in a "Hare's hydrometer." The demonstration is as follows: (*a*) Draw the water halfway up the tubes as shown in figure I. Call attention to the fact that the water levels in the tubes are each at the same height above their respective reservoirs; but that the difference in level of the reservoirs produces a difference in level in the tubes. (*b*) Draw the water up until it just flows over the bend and trickles down into the other leg of the tube (see figure II). In this state the mechanism of the transference of fluid is apparent. The atmospheric pressure tends to support water columns of the same height in each tube, but as one reservoir is higher than

the other a flow obtains. (*c*) Finally the water is drawn up to L (figure II), and the siphon acts quickly.

After the two reservoirs have come to the same level the removal of the block K starts the action in the opposite direction. By alternately inserting and withdrawing K, the siphon may be kept in action as long as desired. Hardwood sawdust or a pinch of black pepper in the water provides particles that are carried along by the current and indicate the flow to the class. Detailed explanation will suggest itself to any teacher.

II. Limit to the siphon's action.

Experiments on the above subject are seldom performed in class except in the case of the mercury siphon under the air pump. But this is not suitable for large classes and the following disposition, while not quite the same, has been found preferable.

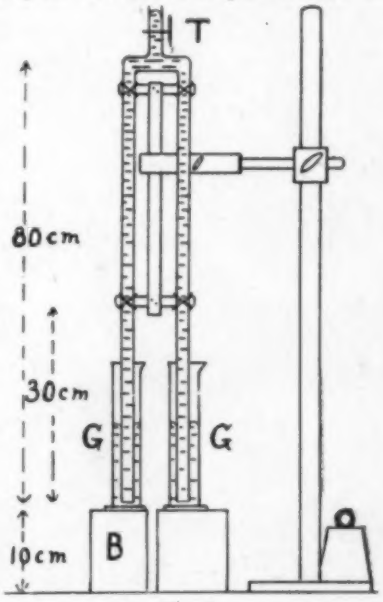


Fig. 3

A tube of the form and dimensions shown in figure III has a well fitting tap at T and is bound to a light wooden frame. Two glass cylinders (GG) (graduated tubes) are filled about eleven centimeters deep with mercury and are set on blocks that hold them about ten centimeters above the lecture table. The siphon is held in a stand at such a height that the open ends of the legs are nearly touching the bottoms of the graduates, as shown. A water pump is connected to the end above T, and the mercury drawn up as far as possible. The whole system is then tipped forward until the mercury fills the whole tube below T. On removing the block sealed with about one centimeter of the fluid. When the apparatus is again set up right (and the pump disconnected), the merucy fills the whole tube below T. On removing the block B and letting the graduate stand on the table, a very rapid siphoning takes place. On replacing the block, the action re-

verses. If both blocks be removed the siphon will not act as the bend is now about seventy-nine centimeters above the level of the reservoirs. Thus the limit of siphoning, in the case of mercury, is actually demonstrated before the class.

By gradually raising one of the cylinders the mercury may be caused to flow across the bend of the tube and pass drop by drop into the other leg of the siphon in exactly the same manner as in part (*b*) of the experiment mentioned above. The comments to the class during these operations are obvious.

TUNGSTEN.

Tungsten has become of renewed interest within the last five years, owing to its wide and successful application for the manufacture of so-called self-hardening rapid tool steels. Its chief ores are wolframite, $FeWO_4$, in which some of the iron is often partly replaced by manganese, hubernite, $MnWO_4$, and scheelite, $CaWO_4$. The ore most suitable for the production of the metal is wolframite. Although found in many places, such as Queensland, New South Wales, Straits Settlements, Bohemia, Cornwall, Spain, the United States of America, etc., it does not occur in large masses, and is often very pockety. In the pure state it should contain over 70 per cent WO_3, rendered the WO_3 soluble as Na_2WO_4, and left the tin behind, it was not until the system of electro-magnetic separation was applied to those mixed ores that the problem was satisfactorily solved. The ores, whether dressed in the ordinary manner or electro magnetically separated, should be as free as possible from tin, arsenic, sulphur, and phosphorus, otherwise these impurities are apt to find their way into the metal made from them. The ore is sold on the basis of its WO_3 contents. At the present moment it is worth in London about 25 shillings to 27 shillings per unit of WO_3—thus a 65 per cent ore is worth about $425 per ton. The prices of ore have varied in an extraordinary manner; at one time it could be bought for $75 per ton, and last year it went as high as $750 per ton. With regard to scheelite, this mineral is a white heavy mineral, named after the Swedish chemist Scheele, who first found tungstic acid in the same. As it is not quite so suitable for the manufacture of tungsten metal, it has a somewhat lower value than wolframite, although, if pure, it contains well over 70 per cent of tungstic acid, not infrequently as much as 74 to 75 per cent. The world's production of wolfram ores is about 3,000 tons per annum, based on 60 to 65 per cent ore. Queensland seems to have been the chief producer in 1904, with 1,538 tons, valued at $808,175; next probably come Spain and Portugal, with about 400 to 500 tons.—*O. J. Steinhart, Trans. Inst. M. and M.*

SOME NOTES ON CHEMICAL LABORATORY EQUIPMENT.

By R. M. Hughes,

Miami University, Oxford, Ohio.

Reagent bottles. While neat looking and clearly labeled reagent bottles are most desirable in the laboratory, those with good labels in the glass supplied by the dealers are rather expensive. If one bottle of such a set is broken, as frequently happens, a second must be ordered or if the missing bottle is replaced by an odd bottle the set will look badly. Fully as neat and as clearly labeled bottles may be prepared much more cheaply as follows: Buy German Tincture bottles—i. e., narrow necked—or salt mouth bottles of the desired size, with well ground stoppers. These can be imported duty free for educational institutions to great advantage. Obtain blank labels 1½ by 2½ inches with a black border. The Dennison Mfg. Co. of Chicago will supply them. Write for the catalogue of the Tablet and Ticket Co. of Chicago and order of them one box containing 1,000 assorted letters and figures size number 1, and one box containing 1,000 letters only, size number 3. The labels are first pasted on the bottles at the desired height. The gummed letters are then stuck on the label and when a number of bottles have been completed the labels are coated with paraffin. For this purpose melt in an evaporating dish 2 parts of soft and 1 part of hard paraffin and heat until it just begins to smoke; then lower the flame and maintain it at that temperature. At a lower temperature the paraffin does not flow freely from the brush, and at a higher temperature it will quickly spoil the brush. Apply the melted paraffin to the label with a flat brush an inch wide, painting it on in even strokes so applied that the paraffin extends about one quarter of an inch on the glass beyond the label. In case the bottle is to contain a strong acid or alkali apply the paraffin in an extra thick coat at the upper edge of the label. Five hundred cubic centimeter bottles labeled in this way by a student assistant at fifteen cents an hour

cost complete about twelve cents each. In case a certain reagent is no longer needed the label is readily scraped off and the bottle may be relabeled as desired. If a bottle of a set is broken it can be at once replaced. All of the reagent bottles at the Massachusetts Institute of Technology are labeled in the way described.

The care of brass apparatus. Brass screw clamps, test tube holders, blow pipes, etc., become badly corroded after a year's use, and often are not fit to reissue to students. All such brass ware may be readily cleaned by immersion in concentrated hydrochloric acid for a few minutes. After all corrosion is removed, or so loosened that it readily washes off, wash the article thoroughly in running water and dry on a wire gauze over a flame. In the case of screw clamps or other apparatus having threads and taps, touch them with a piece of paraffin while they are still hot. The melted paraffin spreads quickly over the article and both lubricates it and protects it from further corrosion. Corroded Bunsen burners that have screw adjustments can be very satisfactorily cleaned in this way and in fact the worst looking brass articles can be made as good as new. I am indebted to Professor J. A. Culler of the department of physics for the suggestion that led to the use of this process for cleaning chemical brass ware.

ON THE DEVELOPMENT OF ELEMENTARY GEOMETRY IN THE NINETEENTH CENTURY.

By G. A. Miller,

University of Illinois.

Under the above title Max Simon recently issued a volume which was published in the form of a report to the German Mathematical Society. Probably no one occupying a chair of mathematics in a German university has taken a more active interest than Professor Simon in the recent reform movements relating to the work of teachers of secondary mathematics. He is earnest in his advocacy of reform, zealous in his application of the history of mathematics to the principle of teaching, and full of that good-humored argument that makes a man acceptable as a speaker in an assembly of teachers.* While his writings are not as free from inaccuracies as one might expect, they contain

*D. E. Smith, Bulletin of the American Mathematical Society, Vol. 13 (1906) p. 139.

many instructive partial truths and are characterized by a boldness of statement which is refreshing and very helpful in the acquisition of some great truths. The main object of the present note is to exhibit some of the conclusions of Professor Simon's report, which seem of especial interest to teachers of elementary geometry.

We first note a remark in regard to popular text-books. On page 33 Professor Simon mentions the work on elementary mathematics by L. Kambly, which went through more than one hundred editions, and he observes that this success is without a parallel, although the work had no scientific value. He adds that the great success was perhaps due to the fact that it had no scientific value. This reminds one of the statement, "Any text-book will be extensively adopted if it is only poor enough," and shows that such remarks are not confined to conditions in this country. On page 42 he calls G. A. Wentworth the American Kambly. An important remark in regard to the use of text-books is found on page 26, where it is stated that it is impossible to make a sufficiently sharp distinction between the teacher and the text-book. For pupils below twelve the use of a text-book is regarded by the author as simply a crime and he thinks that a table of logarithms is the only mathematical book that needs to be in the hands of pupils. The teacher should, as far as possible, present the subject in such a genetic manner that the pupil feels that he has discovered the theorems independently, while a text-book, on the contrary, has to be dogmatic.

In summarizing the changes of methods of instruction in elementary geometry during the nineteenth century the following facts are emphasized: There has been a gradual abandonment of the dogmatic viewpoint in favor of the genetic and this change has been attended by greater emphasis on exercises and constructions. More attention has been paid to intuition as an independent and important factor in teaching geometry, and this gave rise to the so-called fusion of plane and solid geometry, which has been especially emphasized in France by the works of Meray, and has found strong exponents in the other countries. Toward the end of the century a strong wave of critical investigations in regard to the foundations of geometry started in Italy and was strengthened and popularized by Hilbert's *Grundlagen der Geometrie*.* While this has affected the advanced work in geometry

*Translated into English by E. J. Townsend.

more than the elementary, yet it has made itself felt in the latter, and in our country has led to such a work as Halsted's *Rational Geometry.*

The gradual abandonment of Euclid and the arithmetization of geometry are described as follows: The French Revolution which destroyed all authority was also turned toward Euclid. Legendre's Elements arose, perhaps through the influence of d'Alembert, but more in opposition to the important work of the Jesuits, and they were rapidly spread over all the Romanic countries, including Belgium and Holland. The arithmetization of geometry began with these elements and acquired momentum especially through the works of Weierstrass and through George Cantor's theory of point sets (Mengenlehre), which aims to give an arithmetic definition of the purely geometric concept continuity.

The Elements of Legendre (the second Euclid) have not only been conducive to the arithmetization of geometry but they have also contributed much toward the critical study of the axioms of geometry—especially of the one relating to parallel lines and known as the eleventh axiom of Euclid. In the first edition of the Elements (1794) Legendre gave what he considered a proof of this axiom and in the later editions he gave what appeared to him simpler proofs of what is really unprovable. The extensive circulation of Legendre's Elements attracted attention to these efforts to prove one axiom by tacitly assuming another and thus intensified the interest in geometries in which one or more of the axioms of Euclid is not assumed—the Non-Euclidian geometries—whose development is one of the greatest scientific triumphs of the nineteenth century.

In his excellent critique of the fundamental concepts of geometry, Wellstein gives expression to the following ideas relating to this development:* The criticisms of Euclid should have begun with the first definition (the definition of a point as that which has no parts) instead of with the eleventh axiom. This concept of point arises from that of a material point through the process of passing to a limit. One thinks of a grain of sand or some other particle of matter and conceives it as becoming smaller and smaller without limit. During this process it appears to become less and less possible to divide the grain of sand into smaller parts and the concept of point is said to arise as a

*Weber and Wellstein Encyklepadie der Elementa Mathematik, Vol. 2, 1905.

limit of this process and hence as something without parts. This conception of a point is untenable, for it is impossible to conceive of the particle as becoming smaller after it has become so small that it cannot be seen. When this point has been reached we are in perfect darkness since we cannot conceive that the process of becoming smaller and smaller has necessarily an end, but we must believe or postulate that there is some limit beyond which the particle cannot be decreased and which it cannot reach.

To get a clearer idea of the given conception of a point we may think of viewing the particle, which becomes smaller and smaller, through a microscope whose power is continually increasing at such a rate that the particle will appear to keep the same size. Hence, no matter how small the particle has become, the possibility of dividing it remains unchanged, and if we want to associate the concept of point with that of a particle which becomes smaller and smaller it should be clearly observed that this is an act of the will and not one of reason. Hence the first definition of Euclid is based upon the postulate that points exist. They have, however, no spatial existence. It is evident that similar remarks apply to the definitions of line, plane, etc., and we are reminded that nothing is a greater hindrance to a deep understanding than that which appears self-evident at the first glance. As the elements with which one operates in elementary geometry have their existence in postulates, it follows that pure geometry relates only to thinking beings. Fortunately, the trend of the developments has been such as to make the applications to other subjects easy.

In an appendix to his report Professor Simon speaks of the efforts that are being made in Germany to secure a better general understanding of the importance and of the nature of a mathematical training. He says that the chief enemies to a proper appreciation of mathematics are the classical philologists who do not want to be convinced, but are anxious to maintain their position in the educational system. The philologists are, however, not the only ones who have hindered proper mathematical training. Even the children of mathematics—those engaged in technical education, and the natural sciences—have threatened to block the way toward proper mathematical training. The former have gone so far as seriously to demand that only such mathematical problems as present themselves in practice should be considered in the training of students. They were even so kind as to offer to prescribe the problems since they themselves were unable to solve them. These unfortunate tendencies have been largely checked by the wise and untiring efforts of Professor Klein, and still more by the power of the facts which is stronger than that of men.

EDUCATIONAL PROGRESS OF THE SOUTH.

By Charles A. Shull,

Kentucky University, Lexington, Ky.

The splendid educational progress of the Southern states during the last few years has never been equaled by any other people in the history of mankind, and is an index of the enthusiastic interest which has been and is being aroused among the people of this region in all matters pertaining to thoroughness of education and the elevation of intellectual life and ideals.

It is most encouraging to know that the improvement has begun at the foundation of our educational system, with the public schools, and that it forms therefore a permanent basis from which to build. New schoolhouses have been erected, school terms have been lengthened, laws governing child labor and compelling attendance in school have been enacted, better training for the teacher has been provided in state normal schools, and new sources of revenue have been opened up to supply funds to carry the work forward to an immediately successful issue.

This elevation of the character of the work done in the public schools must naturally be felt in the high schools which are being multiplied in practically every Southern state. A higher grade of work can be established and maintained. It is possible, therefore, to strengthen the college curriculum and increase the efficiency of every higher institution of learning in proportion as the character of the work in each unit of the system is elevated.

So far Kentucky has had but a very small part in this forward movement, as compared with the splendid achievements of the Carolinas, Alabama, Georgia, Mississippi, and Texas. Nevertheless, some things have been done to improve conditions —the length of the school term has been increased from five to six months; state normal schools have been established at Richmond and at Bowling Green; school improvement leagues have been organized; and most important of all, perhaps, the conditions now obtaining in the state are being investigated and the truth made known to every citizen of the commonwealth. Much of what has been done we owe directly to the efforts of the enlightened women of Kentucky, not to the wisdom of its legislators.

It is hoped that public opinion which is now more fully aroused may sweep Kentucky onward into the full current of educational progress, and make it as in its earlier history the first rather than well-nigh the last state in the Union in the achievement of truly high standards and ideals of education.

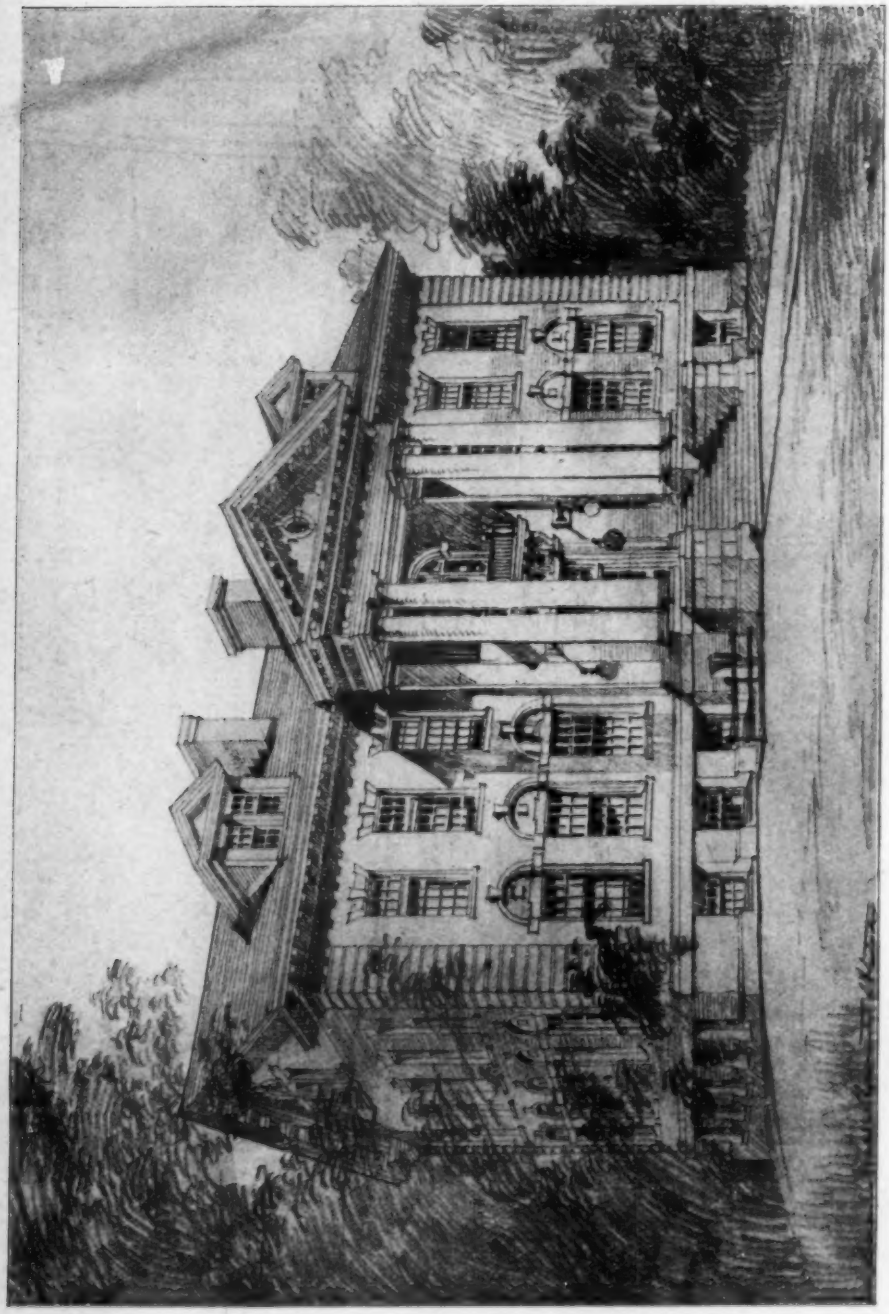
The disciplinary and cultural value of the study of the natural sciences was for many years unrecognized in our higher institutions of learning, even after it had won a permanent and ever increasing place of importance in the colleges and universities of other regions. The advancement made in this direction may be exemplified by the improvements now in progress at Kentucky University, Lexington, Ky.

A new building of brick and stone to be devoted exclusively to the natural sciences, chemistry, physics, zoölogy, and botany, is being erected, and will be completed before the close of the current year. An idea of the general appearance of the structure, which is 90 feet long and 50 feet wide, may be gained from the accompanying plate, which was made from a photograph of the architect's drawing.

The first floor is occupied by three chemical laboratories, one large room to accommodate the classes in general inorganic chemistry, while two smaller laboratories will be used by students of qualitative and quantitative analysis respectively. In connection with these laboratories are balance rooms, storage rooms, etc., with a good equipment of chemicals and apparatus for all phases of the work.

The second floor is occupied by the physical laboratory, a class room for students of chemistry and physics, and a large, well illuminated museum. The laboratory is to be fully supplied with apparatus of most modern and improved types for the study of molar, molecular, and ethereal dynamics, and the class room will be provided with projection apparatus for the illustration of lectures. In the museum display cases and tables will be installed, and the collections of minerals, birds, and archaeological specimens will be removed from their present overcrowded quarters and arranged systematically, so as to be of the greatest possible advantage to the students. Efforts will be made greatly to enlarge the various collections.

The entire third floor is devoted to zoölogy and botany. A large laboratory for students of general invertebrate and vertebrate anatomy will be equipped with a full supply of dissecting



and compound microscopes, and with microtomes, paraffin bath, incubators, and a complete series of biological reagents for preparation of histological and embryological material. An aquarium will make it possible to keep living material on hand in the laboratory.

Opposite the zoölogical laboratory is the botanical laboratory, and a class room for students of biology. This laboratory will be equipped with all the apparatus and reagents necessary for elementary and advanced courses in plant physiology, plant physics and chemics, morphology and microscopic anatomy. It will also contain an aquarium.

The class room will be supplied with projection apparatus, lantern slides and stereographs, wall charts, demonstration materials, and other accessories conducive to interesting and thorough work.

The erection of such buildings as this by the various colleges and universities of the South, and the endowment of the departments with funds sufficient to equip and maintain them in a high state of efficiency, marks a long step forward in the teaching of the natural sciences. It seems to me a safe prediction that in a few years our institutions will not be excelled in equipment for scientific investigation, neither as to men nor as to apparatus, by those of any other region.

The vast natural resources of the Southern states, as yet but slightly developed, the rapid increase in land values, the growth of railways, and the rapid development of manufacturing enterprises, which tends to keep the wealth of the land at home, all are contributing to the unparalleled prosperity of the region. And public opinion, now wide awake to the advantages of better education, is demanding the use of this wealth in the establishment of the best system of public instruction in the world. The South, with its new found wealth and energy, is our best hope for the future of educational progress in America.

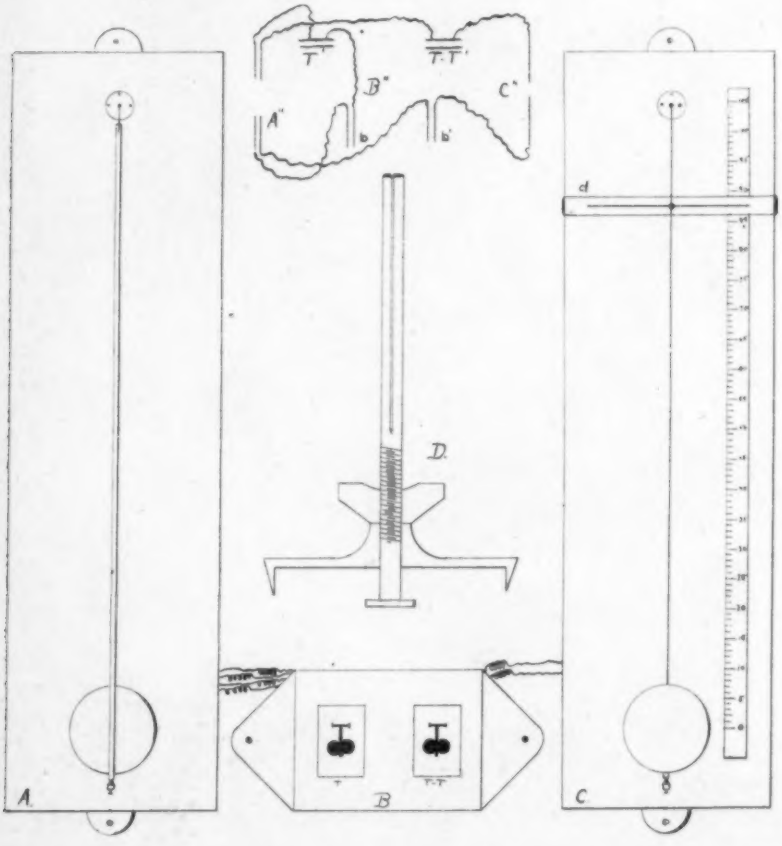
AN APPARATUS FOR DERIVING LAWS OF PENDULUMS.

By Ellison L. Ross,

University of Nebraska.

The apparatus consists essentially of two pendulums—one a second's pendulum and the other of variable length, which are electrically connected to two telegraph sounders in such a manner that one of the sounders will tick every time the second's pendulum passes the central point of its vibration. The other sounder will tick only when both pendulums are at the central points of their vibrations at the same instant.

In the accompanying drawings A represents the front view of the second's pendulum, A′ the side view of the same. This pendulum differs from the common pendulum only in that it has two rods supporting the bob instead of one. These two rods



are electrically insulated from each other with hard rubber. The mercury cups are on separate brackets, one curving around the other. Thus A may carry two independent currents which may be closed and broken simultaneously by the vibration of the pendulum.

Figure C represents the pendulum whose length may be varied. The bob of the pendulum is of the same shape and size as that of the other pendulum and holds a copper rod with a platinum or silver point for the mercury contact. The suspension of the bob is by very thin, clock-spring-steel strips. The manner of suspension is shown in Figure C'. Figure C' is the side view of C. The length of the pendulum is varied by a clamp d, d', and D which slides in a slot in the standard and is tightened with a thumb-screw. The length of the pendulum may be read from the scale as shown in the figure. Electrical connection is made at the point of suspension at the bracket of the mercury cup.

Figure B represents a box which is just large enough to hold two small common dry cells. On the outside of the box are fastened two telegraph sounders.

The electrical connection between the pendulums, batteries and sounders is shown in Figure A″B″C″. Part A″ represents the course of the two currents through pendulum A. Part B″ shows the connections that are made in the box connecting sounders T' and T-T', and cells b and b'. Part C″ represents the course of the current through pendulum C.

The whole apparatus is meant to be screwed to the wall.

When the apparatus is in order and both pendulums are in motion, the sounder T will tick every time the lower end of

second's pendulum passes through the mercury, but sounder T-T will tick only when the ends of second's pendulum and variable pendulum are passing through the mercury cups at the same time. For two operators the derivation of the Law is simple. The length of the variable pendulum is determined and both pendulums are set in motion. When sounder T-T ticks one operator begins to count the ticks of T and the other the number of vibrations of pendulum C, both stopping when sounder T-T ticks again. The length may now be changed and the operation repeated.

With this apparatus constructed as described, there will be a possible source of error, but probably small enough to be negligible. This will be due to the fact that the suspension of the variable pendulum, being steel, will not be perfectly pliable, thus making the length of the pendulum not exactly the distance between the center of mass of the pendulum and the lower edge of the clamp. This error may be overcome by using silk cord for suspension and connecting the bob and point of suspension with a fine slack insulated copper wire for carrying the current. But if the apparatus is to be subject to hard general usage, the latter construction is impracticable, as the author has learned from experience.

The essentials of this apparatus were put together by the author while teaching Physics at North Platte, Neb., and while the construction was more crude than that described, High School students had no trouble in deriving the Law comparatively accurately in a short time.

THE TEACHING OF MATHEMATICS IN CALIFORNIA SECONDARY SCHOOLS.*

By Charles A. Noble,
University of California.

It is a regrettable fact that a considerable proposition of the students of the University of California enter college without mathematical training sufficient to enable them to profit by a first course in analytic geometry, or in college algebra; that is, without adequate familiarity with the fundamental processes of algebra, or ability to handle the quadratic equation intelligently. The reasons for this deficient training are not far to seek. In many high schools the pupil finishes his algebra with the second year.

*Read before the California Association of Science and Mathematics Teachers.

During the last two years his acquirements become unavailable—he gets rusty—and he is certain to be handicapped upon entering classes where a working knowledge of elementary topics is presupposed. The obvious remedy for this deficiency is to require every pupil, who looks toward engineering study, or toward anything in mathematics beyond the minimum requirement of a college course, to devote a portion of his last high school year to algebra.

Our matriculants betray, however, another and more serious form of deficiency, which results from defective training. They have not been taught to think. They have not learned that algebra has theorems that must be proved by courses of close reasoning. There is a tendency among high school teachers to develop, in pupils, mechanical skill which is not guided by reason—to drill them in working examples without inquiring into the significance of the processes employed. Far be it from me to urge the presentation of the beginnings of algebra exclusively by means of proved theorems. It is not necessary that every step be demonstrated at first. The essential thing is to train the pupil to appreciate the nature of demonstration, and the ultimate necessity for it. The following suggestions may be serviceable, as indicating the amount of theory that, in the judgment of the writer, can profitably be given to the high school pupil during three to four terms of work.

The beginnings of algebra should be presented in analogy with, and in extension of, arithmetic; but the pupil should be shown that there is a real extension, that algebra is not mere jugglery with symbols. Above all, the pupil must not be taught what is false, and must be unlearned; e. g. that the laws of operation are axiomatic, or that there are no real numbers except integers and rational fractions.

If the beginner early grasps the notion that subtraction and division are inverse operations, some future difficulties will be forestalled, as, for example, the inadmissibility of division by zero. If he learns to realize the difference between arithmetic value and algebraic form, it should not be difficult for him to appreciate the rational, as distinguished from other operations, and so gain the valuable notions of rational integral and rational fractional functions.

The young pupil learns to solve equations with readiness, but he is apt to remain in ignorance of the meaning of the solution,

and the significance of the process wihch yields it. He doesn't realize that he has to deal with a *function*, which vanishes for a particular value of the variable. He seeks an "answer," concerning the legitimacy of which he remains in suspense, until he establishes its correctness by actual substitution, which he calls proof. If the pupil is taught the significance of equivalent equations—of reversible and non-reversible steps, he must recognize that, barring errors of calculation, legitimate reasoning cannot but yield the correct result. This caution has especial significance in the solving of simultaneous equations, and in cases where inadmissible results may appear.

A proof of the commutative, associative, and distributive laws would be lost on the majority of pupils, during the early months of the study of algebra; but it is easy to cite examples to show that these laws are not valid everywhere, and thus to convince the pupil that proof is not superfluous; and, at some time in the course, a proof of these laws, as well as other explicitly omitted demonstrations, should be offered for the benefit of the few who can appreciate them. The proofs of the laws of operation with fractions are sufficiently easy to warrant their being given early in the course.

Considering the time devoted to factoring, pupils show singular lack of understanding. The average freshman will factor x^2-4 readily, but will declare that x^2-5 is unfactorable. The fault lies in requiring pupils to hunt for factors, without giving them clear-cut criteria. They do not realize that an expression may or may not be factorable, according to the demands made as to the form of the factors. Some if the current text-books ignore the valuable method of factoring a quadratic trinominal by transforming it into the difference of two squares. Pupils are taught to factor such expressions by inspection and trial, a plan unobjectionable in easy examples, but inadequate as a general method.

In view of the number of essential topics in algebra that must be offered to the young learner, too much time is devoted to evolution. The relative importance of square root is small, of cube root almost nil. They are detached subjects, interesting enough in themselves, but not necessary to mathematical progress, nor very valuable as discipline.

In a number of text-books, the chapters on irrational numbers and the theory of indices are introduced so early that many

teachers, especially those who follow the book literally, lead the pupil into an extended discussion of these notions the first year. These subjects are difficult, and had better be relegated to the second course in algebra. In many of the texts, too, the presentation here is faulty; e. g. the law $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ is sometimes announced as a general one, or one that the author has established. One needs merely to put $n=2$ and $a=b=-1$ to exhibit the error. In studying the theory of indices, as based on the fundamental index laws, pupils commonly have difficulty in enunciating the theorems they prove. A statement is likely to lack hypothesis, to consist entirely of inference. This is merely an exaggerated form of the difficulty pupils generally have, in stating the assumptions of an algebraic theorem. The fault lies partly in the abstruseness of the subject, and partly in the tendency of teachers to follow the time-honored custom that would confine formal theorems to geometry.

Thus far, these remarks have been confined mainly to infelicities in the teaching of those chapters of algebra that high school pupils usually study during a first course. The University of California explicitly requests that, in the teaching of the customary second course, emphasis be laid on the intelligent demonstration of principles—not implying, by this, that the pupil's acquirements in the preliminary course should be based on dogmatic statements of text-book or teacher, but merely recognizing that, in the pupil's development, the ability to comprehend and reproduce rigorous proof is of gradual growth. Proceeding to those topics that are usually studied in a second course, the writer would note a few points where the pupil's training could be bettered.

In solving quadratic equations, as in the case of those of the first degree, the pupil acquires mechanical dexterity, but is apt to lack ability to assign reasons for his deductions. He will bring the quadratic to the form $a(x-\alpha)(x-\beta)=o$ and say that, since either $x-\alpha$ or $x-\beta$ must vanish, therefore the roots are α and β. He overlooks the equally pertinent fact that, if either $x-\alpha$ or $x-\beta$ is zero, the above product must vanish. He is not led to distinguish between necessity and sufficiency.

If the pupil is studying simultaneous equations of higher degree, he is sometimes allowed to overlook the artificial nature of those sets of equations which he is asked to solve by quadratic methods. He is permitted to believe that every pair of equations

in x and y is solvable. Furthermore, many pupils fail to see the necessity of combining each x with its appropriate y—they fail to recognize that it takes a *pair* of numbers to effect a solution. Once the pupil realizes, however, that the process of solving a group of simultaneous equations consists, essentially, in successively replacing one group by an equivalent group of simpler equations, he is likely to escape the danger of coming out, at the end of his calculation, with a list of x's and y's before which he stands helpless until he has tested them all. The use of graphical methods is almost indispensable, here, in ordering the pupil's commonly confused notions.

The teaching of mathematical induction seems peculiarly difficult. Even after the pupil has been convinced that no general law can be established by special cases, he persists in going astray. He assumes the law to hold for n cases, and shows that it then holds for $n+1$ cases. There he stops, thinking the proof complete. He fails to identify n with a known case, so as to eliminate the assumption.

It is perhaps unfortunate that the abstract notions of limit and infinity should present themselves to high school pupils; but these notions are indispensable to an adequate discussion of the trigonometric functions, or of the quadratic equation. The definition of limit in the text-books is usually adequate for elementary needs, but the illustrations are frequently misleading, in that the pupil is persuaded that the function is always different in value from the limit, even that it is always smaller than the limit; and these false notions are sometimes supported by the series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$, the sum of n terms of which is, to be sure, always different from, and smaller than, the limit of that sum, viz., unity. In the progression $\frac{3}{2}-\frac{3}{4}+\frac{3}{8}-+\ldots$, however, the sum of n terms is alternately larger and smaller than the limit. Neither teachers nor text-books always distinguish between the limit of $f(x)$ when x approaches a, and the actual value of $f(x)$ when x equals a. If, for example, $f(x)=\frac{x^2-4}{x-2}$, then $\lim_{x=2} f(x)=4$, but $f(2)$ has no definite meaning. On the other hand, $\lim_{x=1} f(x)=3$ and $f(1)=3$ also. If however, one puts $f(x)=\sin\frac{1}{x}$ and allows x to approach zero, $f(x)$ assumes every real value between $+1$ and -1, over and over again, without approaching any fixed limit. Thus neither $\lim_{x=0} f(x)$ nor $f(0)$

exists. This well-known example illustrates the possibility, that the usual text-book definition of limit may be satisfied, under circumstances where, nevertheless, a limit does not exist. The difference $|\sin\frac{1}{x}-a|$, where a is any fixed number between $+1$ and -1, will become smaller than any preassigned positive number e, as x approaches zero, but it does not *remain* smaller than e.

The notions that the high school pupil gets from class discussions of infinity are not infrequently grotesque. Infinity becomes for him a constant, so large that it must be mentioned with awe; or a mysterious something to haggle over, and to furnish paradoxes. This seems unnecessary. The situation that arises, wherever this concept presents itself, is the following: Under certain circumstances, a variable number, x, increases in such a way that it can exceed, in absolute value, any preassigned constant. This fact is expressed conventionally, by saying that x becomes infinite, under the assigned circumstances. The example most familiar to the pupil is the trigonometric ratio, tan θ. If the angle θ becomes $\frac{\pi}{2}$, the tangent does not assume a definite value—on the contrary it ceases to be a definite number, the defining ratio loses its meaning. But for all values of $\theta\pm\frac{\pi}{2}$ tan θ has a definite meaning, and as θ approaches $\frac{\pi}{2}$, the variable, tan θ, may be made to exceed, in absolute value, any constant previously agreed upon. Current text-books are partly to blame for the pupil's hazy notions, when the concept infinity is involved; teachers are to blame for allowing errors of the text to pass unchallenged. In one text, we read that x becomes infinite when it exceeds "any conceivable number"—a manifest contradiction. From another text, we learn that infinity is the number $\frac{1}{0}$, in spite of the author's earlier statement that division by zero is meaningless.

Teachers fail to make it clear that, under the above definition, infinity must be a variable. The necessity for the "preassigned" number of this definition being a constant, is likewise overlooked. E. g. when $x=o$, $\frac{1}{x}$ becomes infinite; but it cannot be made to exceed $\frac{1}{x^2}$. Yet $\frac{1}{x^2}$ would serve perfectly as the comparison number of some texts and of many teachers—it is just as "conceivable" as $\frac{1}{x}$.

Ambitious teachers sometimes include, in the school curriculum, one of the courses in advanced algebra (numbered $12a^3$, $12a^4$ in the matriculation list of the University of California), although the topics concerned are too difficult for the majority of pupils in the average school. A four years' course in mathematics reads well in the school announcement, and the well-trained teacher finds the exposition of the theory of equations, convergency of series, continued fractions, etc., a pleasurable occupation. But in some cases the intellectual health of the school is sacrificed by encouraging, or permitting, pupils to take these subjects, who are unable to master the difficulties involved. If classes are to be formed in $12a^3$ or $12a^4$, they should be of selected pupils, of the senior year; and unless the courses can be given in a thoroughgoing way, they should not be given at all. The writer's personal conviction is, that the University of California should withdraw $12a^3$ and $12a^4$ from its published list of matriculation options, and so discourage their being attempted, except in response to a normal demand.

In conclusion, the writer would like to offer a few suggestions to inexperienced teachers, as to the conduct of recitations. Every teacher should make thoughtful preparation for each class exercise. However well he may know his subject, he owes it to the class, that he should organize his material, and so plan the recitation, that appropriate stress may be laid, and the greatest profit accrue to the greatest number. It is a common mistake for the novice to look upon a recitation primarily as a test of the individual who is reciting. This is not teaching. The reciting pupil is rather a foil for the instruction of others, himself learning, at the same time, oral exposition and all that goes with it. This means that only those propositions should come up for discussion that are most significant, or that involve difficulties which must be cleared away. It means that the reciting pupil should have the attention of the class—not merely that he should talk to the teacher while his neighbors are getting ready for their turns. The recitation period is short. It belongs to the class as a whole, and the teacher should see to it that every moment is profiting the largest possible number. No part of it should be devoted exclusively to individual instruction. If a pupil needs something that nobody else wants, he might get it from the teacher at another time or from some fellow pupil. Too much time is consumed, during recitation hours, in writing upon the

blackboard. It is characteristic of a large number of teachers, to start a class exercise by sending a troop of pupils to the board, where from a third to a half of the period is spent in writing a demonstration or the solution of a problem. Perhaps those in their seats are given a problem to keep them busy. This is virtually wasting time for all in the class. When those at the board have finished, they are required to read aloud what they have written—a deadening process, because it lacks spirit and spontaneity. How much more interesting and profitable it would be, if the pupil, before reciting, should put upon the board merely a figure, or the skeleton of a demonstration; or, still better, why not have a pupil start with a clean board, and put down figures and equations as necessity for them develops? This would be the ideal of a comprehending and instructive exposition. It would serve toward overcoming what we all recognize as the most regrettable lack in our pupils—the inability to express themselves in pertinent, intelligent English.

GIFTS.

To choose an appropriate gift—one to be received with genuine pleasure—is truly an accomplishment. Perhaps a suggestion will be of assistance to you before making your purchases for the holiday season. Have you ever considered that an up-to-date unabridged dictionary is a gift to be longer enjoyed, longer treasured, and of more constant service to the recipient than any other selection you may make? The One Great Standard Authority is Webster's International Dictionary, published by G. & C. Merriam Co., Springfield, Mass. It is recognized by the courts, the schools, and the press, not only in this country, but throughout the English-speaking world as the highest triumph in dictionary making. It is the most choice gift. Get the best.

GRAPHICAL ILLUSTRATION OF CONVERGENCE OF SERIES.*

S. EPSTEEN AND F. W. DOOLITTLE,

University of Colorado.

Experience shows that beginners find the chapter on Convergency and Divergency of Series one of the most difficult in the entire subject of elementary algebra.

The following graphical presentation has proved very helpful in the Freshman Engineering Algebra of the University of Colorado. *The treatment is not, and could not be made, rigorous;* it is intended merely as illustrative, in order that the students may have before them the concepts and theorems in pictorial as well as in algebraic form.

Definitions and examples are given in order that this paper may be a self-sufficient chapter for teachers who deem a greater amount of rigor not desirable when covering the subject for the first time. Our own method consists in having this graphic presentation supplement rather than supplant the text-book.

Infinite Series. If $u_1, u_2, u_3, \ldots\ldots u_n$ denote any given never ending sequence of numbers, the expression $u_1+u_2+u_3+\ldots\ldots\ldots+u_n+\ldots\ldots$ is called an infinite series.

For $u_1+u_2+\ldots\ldots$ we may write Σu_n, read "sum of u_n to infinity."

The series is called real when all its terms $u_1, u_2, u_3, \ldots\ldots\ldots$ are real, positive when all its terms are positive.

A series is often given by means of a formula for its nth term, u_n. Thus if $u_n=\frac{\sqrt{n}}{n+1}$, the series is $\frac{\sqrt{1}}{2}+\frac{\sqrt{2}}{3}+\frac{\sqrt{3}}{4}+\ldots\ldots\ldots+\frac{\sqrt{n}}{n+1}+\ldots$

EXAMPLES.

Write down five terms of the series whose nth terms are,

1. $u_n=\frac{1}{n}$; $\quad u_n=\frac{1}{n^2}$
2. $u_n=\frac{1}{n(n+1)}$; $\quad u_n=\frac{1}{(n-1)n}$
3. $u_n=\frac{2n-1}{(n+1)\ (n+2)}$; $\quad u_n=\frac{1}{\sqrt{n^3}}$
4. $u_n=\frac{1}{na+b}$; $\quad u_n=\frac{1}{1+x^n}$
5. $u_n=\frac{x^n}{n!}$; $\quad u_n=\frac{n}{1+n\sqrt{n}}$

*Read before the Colorado Mathematical Society, Denver, Colorado, May 11, 1907.

Sometimes such a formula is indicated by writing three or four terms of the series. Thus in the series $\frac{1}{2}+\frac{1\cdot3}{2\cdot4}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}+\cdots$ we have $u_n=\frac{1\cdot3\cdots\cdots(2n-1)}{2\cdot4\cdots\cdots 2n}$

EXAMPLES.

Write the nth term, given the series,

6. $\frac{2}{1\cdot2}+\frac{2^2}{2\cdot3}+\frac{2^3}{3\cdot4}+\cdots$

7. $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots\cdots$

8. $\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\frac{4^4}{5^5}+\cdots\cdots$

9. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots\cdots$

10. $2+\frac{3}{4}+\frac{4}{9}+\frac{5}{16}+\frac{6}{25}+\cdots$

Graphical representation of series. On cross section paper we mark two rectangular axes, OX and OY. On OY lay off u_1, on the next ordinate (distance between ordinates = 1) lay off u_2, on the third ordinate lay off u_3, etc. We join these points by a smooth curve as in Fig. 1.

On OX let one small space represent unit length. Then S_1=area of first rectangle, i. e. $l.u_1$; S_2=area of first two rectangles, i. e. $l.u_1+l.u_2$; S_3 =area of first three rectangles, etc. The sum of the series, then, is the area bounded by OX, OY, and the curve.

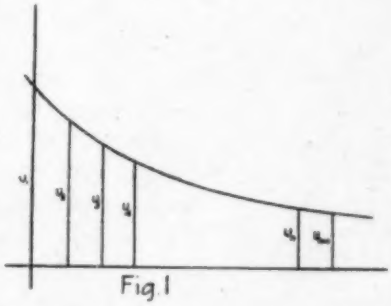


Fig. 1

EXAMPLES.

11. Plot the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\cdots$ as in Fig. 1. (Directions. On the vertical axis, OY, let 20 small spaces represent the length 1. On OY, $u_1=1$. Represent this by twenty small spaces. On the next consecutive ordinate lay off $u_2=\frac{1}{2}$, i. e., 10 spaces. On the third ordinate lay off $u_3=\frac{1}{4}$, i. e., 5 spaces, etc.)

Note that the series is a geometric progression whose common ratio is $\frac{1}{2}$. The sum to infinity is therefore 2.*

*If a is the first term, r the common ratio, n the number of terms, of a geometric progression, then $S_n=\frac{a(r^n-1)}{r-1}$. When r is less than 1, as n approaches infinity, the sum approaches $S=\frac{a}{1-r}$.

12. Plot the series, (*a*) $1+1+1+1+1+\ldots\ldots$ (*b*) $1+1\frac{1}{2}+2+2\frac{1}{2}+3+\ldots\ldots$

Evidently the areas bounded by the axes and these lines are infinite. The sums of the series are therefore infinite.

(c) $1-1+1-1+1-1+$ is indeterminate for S takes successively the values 1, 0, 1, 0.

Convergence and Divergence. Let S_n denote the sum of the first n terms of the series $u_1+u_2+u_3+u_4+\ldots\ldots$ so that $S_1=u_1$ $S_2=u_1+u_2$, $S_3=u_1+u_2+u_3$, that is to say, the area of the first n rectangles bounded by the curve, OX and OY. As n increases, S_n, the area of the rectangles,

1. May approach some finite limit.
2. May approach infinity.
3. May be indeterminate.

In the first case the series $u_1+u_2+u_3+\ldots\ldots$ is said to be convergent, and limit S_n (written $\lfloor S_n$) is called its sum. In the second case the series is said to be *divergent.**

When $u_1+u_2+u_3+u_4+\ldots\ldots$ is convergent we may represent its sum $\lfloor_{n=\infty} S_n$, by S and write $S=u_1+u_2+u_3+\ldots\ldots$ i. e. we may regard the series as another expression for the definite number S. Thus the geometric progression $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\ldots$ is *convergent*, for the sum is 2. It will be seen from the figure of Ex. 11 that the area of the first rectangle is 1, of the first two 1½, of the first three 1¾, of the first four 1⅞, etc. Although the curve never actually closes on the X axis, it is evident that after a while it hugs it so closely that the successive rectangles become exceedingly small. The series of Ex. 12*a* and 12*b* are divergent because the sums are infinite, and in (12*c*) it is indeterminate. In determining whether a series is convergent or divergent, a finite number of terms may be neglected. For instance in Ex. 11 if the first two rectangles are omitted the remaining area is finite (½) i. e. $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\ldots$ is still convergent. In Ex. 12, (*a*) and (*b*), if the first two rectangles are omitted, the area still remains infinite.

Theorem 1. If $u+u_2+u_3+u_4+\ldots\ldots$ is a convergent series having the sum S, and c is any finite number, then $cu_1+cu_2+cu_3+cu_4+\ldots\ldots$ is a convergent series having the sum cS.

The truth of this proposition is apparent from Ex. 13. Plot on cross section paper (on same axes, preferably in different

*A series such as $1-1+1-1=1-1+$, whose sum is indeterminate is sometimes called divergent.

colors) the following series: $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\ldots\ldots$
$3+\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+\ldots\ldots$, $5+\frac{5}{2}+\frac{5}{4}+\frac{5}{8}+\frac{5}{16}+\ldots\ldots$
If the series $u_1+u_2+u_3+u_4+\ldots\ldots$ is divergent the series $cu_1+cu_2+cu_3+cu_4+\ldots\ldots$ is also divergent.

Ex. 14. Plot on cross section paper, as in Ex. 13, the series
$1+1+1+1+1+\ldots\ldots$,
$3+3+3+3+3+\ldots\ldots$,
$5+5+5+5+5+\ldots\ldots$

Theorem 2. A series is convergent if all its terms, or all after a finite number, are less than the corresponding terms of a series known to be convergent. (Verify this by means of figures 2 and 3.)

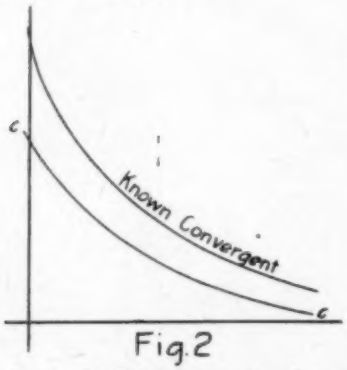


Fig. 2

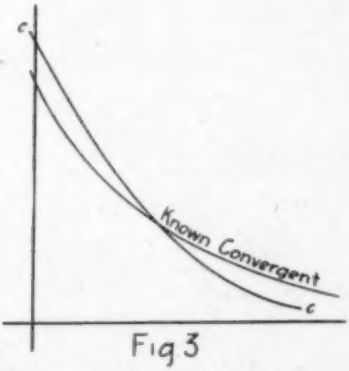


Fig 3

Clearly in Figs. 2 and 3 the series c are convergent.

Theorem 2a. A series is divergent if all its terms, or all after a finite number, are greater than the corresponding terms of a series known to be divergent. (Verify this by means of figures 4 and 5.)

It is obvious in the Figs. 4 and 5 that the series D are divergent.

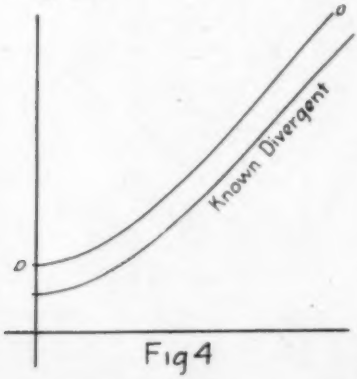


Fig 4

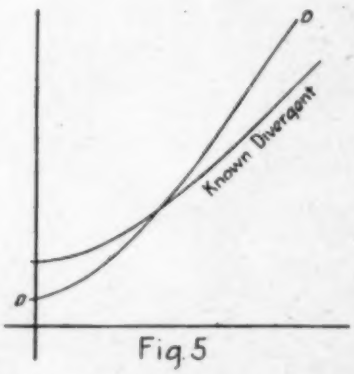


Fig. 5

Theorem 3. (1) The series $S=1+\frac{1}{2^m}+\frac{1}{4^m}+\frac{1}{8^m}+\frac{1}{16^m}+\ldots$ is convergent when $m>1$; (2) It is divergent when $m\leq 1$.

(1) When $m>1$.

We have

$$1=1$$

$$\frac{1}{2^m}+\frac{1}{3^m}<\frac{2}{2^m}$$

$$\frac{1}{4^m}+\frac{1}{5^m}+\frac{1}{6^m}+\frac{1}{7^m}<\frac{4}{4^m}$$

$$\frac{1}{8^m}+\frac{1}{9^m}+\frac{1}{10^m}+\frac{1}{11^m}+\frac{1}{12^m}+\frac{1}{13^m}+\frac{1}{14^m}+\frac{1}{15^m}<\frac{8}{8^m}$$

By addition,

$$1+\frac{1}{2^m}+\frac{1}{3^m}+\frac{1}{4^m}+\ldots\ldots<1+\frac{2}{2^m}+\frac{4}{4^m}+\frac{8}{8^m}$$

But the last series is a geometrical progression whose ratio $r=\frac{2}{2^m}$.

Since $m>1$, $r=\frac{2}{2^m}<1$, and therefore the sum $\frac{a}{1-r}$ becomes

$$\frac{1}{1-\frac{2}{2^m}}=\frac{2^{m-1}}{2^{m-1}-1}$$

Since $S<\frac{2^{m-1}}{2^{m-1}-1}$ it is convergent.

(2) When $m\leq 1$.

When $m=1$, the series becomes

$$1+\tfrac{1}{2}+1/3+\tfrac{1}{4}+1/5+\ldots\ldots$$

Group the terms thus,

$$1+\tfrac{1}{2}+(\tfrac{1}{3}+\tfrac{1}{4})+(\tfrac{1}{5}+\tfrac{1}{6}+\tfrac{1}{7}+\tfrac{1}{8})+\ldots\ldots$$

It is seen that each group is greater than $\frac{1}{2}$.

Hence the series is greater than

$$\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2}+\ldots\ldots$$

and is therefore divergent.

When $m<1$, each term of the series after the first is greater than the corresponding term of the divergent series

$$1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{5}+$$

and therefore by theorem 2a the series is divergent.

EXAMPLES.

Determine whether the following series are convergent or divergent.

13. $1+\frac{1}{2}!+\frac{1}{3}!+\frac{1}{4}!+\frac{1}{5}!+\frac{1}{6}!+\ldots\ldots$

Suggestion: Compare with $1+\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{8}+\ldots\ldots$

14. $1+3+\frac{3^2}{2!}+\frac{3^3}{3!}+\frac{3^4}{4!}+\frac{3^5}{5!}+\ldots$

Solution: After the fifth term each term is less than the corseponding term of the geometrical progression,

$$\frac{3^5}{4\cdot4}+\frac{3^6}{4\cdot4\cdot4}+\frac{3^7}{4\cdot4\cdot4\cdot4}+\ldots\ldots$$

whose ratio is $\frac{3}{4}$. Therefore the series is convergent. (Plot both series.)

15. $\frac{1}{n^3}$.

16. $\frac{n+1}{n^2}$ **Suggestion: Plot this series; also the series** $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots\ldots$

If $\lim_{n=\infty}\frac{u_{n+1}}{u_n}=1$, **we say that the curve *tends* to become parallel to the X axis. Thus, in the series** $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots\ldots$

$u_{n+1}=\frac{1}{n+1}$, $u_n=\frac{1}{n}$, hence $\frac{u_{n+1}}{u_n}=\frac{1}{n+1}\div\frac{1}{n}=\frac{n}{n+1}=\frac{1}{1+\frac{1}{n}}$

$\therefore \lim_{n=\infty}\frac{u_{n+1}}{u_n}=1.$

If we plot the series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots\ldots$, we see that the curve tends to become parallel to the X axis. The area is not bounded on the right side and as we have seen above, the series is divergent, that is, the sum is infinite. If $\lim_{n=\infty}\frac{u_{n+1}}{u_n}<1$ the curve TENDS TO CLOSE or IT TENDS TO MEET THE X AXIS.

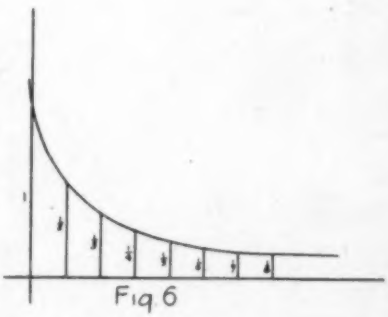


Fig 6

Theorem 4. A series is convergent if after a finite number of terms each term divided by the preceding is <1 and $\lim_{n=\infty}\frac{u_{n+1}}{u_n}<1$ **(i. e. if the curve tends to close).**

Ex. 17. Explain how the following two curves illustrate the statement of this theorem.

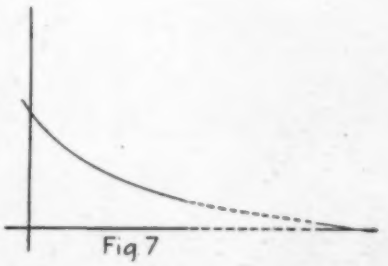
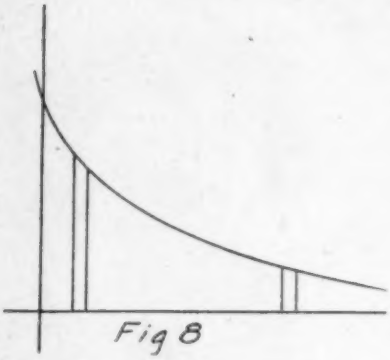
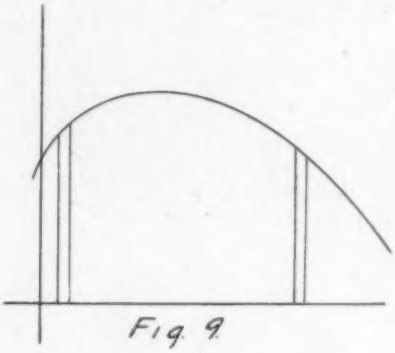


Fig 7





Let the series be

$$S= - - - + u_a + u_{a+1} + u_{a+2} + u_{a+3} + - - - - - \quad (1)$$

$$= - - - - + u_a\left(1 + \frac{u_{a+1}}{u_a} + \frac{u_{a+2}}{u_a} + \frac{u_{a+3}}{u_a} + - - - -\right) \quad (2)$$

$$= - - - + u_a\left(1 + \frac{u_{a+1}}{u_a} + \frac{u_{a+1}\cdot u_{a+2}}{u_a . u_{a+1}} + \frac{u_{a+1}.u_{a+2}.u_{a+3}}{u_a.u_{a+1}.u_{a+2}} - - - -\right) \quad (3)$$

Now if the ratio, r, of each term of (1) to the preceding is less than 1, even when $n=\infty$ (i. e. if the curve tends to close) we have from (3) $S \leq - - - - u_a(1+r+r^2+r^3+ - - -)$ and therefore

$$S \leq - - - - u_a \frac{1}{1-r}$$

Note. If $\lim_{n=\infty} \frac{u_{n+1}}{u_n} = 1$ a special investigation is needed.

Thus $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+ - - -$ is divergent while

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}$$

is convergent, although in both cases $\lim_{n=\infty} \frac{u_{n+1}}{u_n} = 1$

Ex. 18. Shown by diagram: If from the beginning or after a finite number of terms the ratio of each term to the preceding is equal to or greater than 1, the series is divergent.

Theorem 5. A series of numerically decreasing terms which are alternately plus and minus is convergent.

Let the series be

$$a - b + c - d + e - f + \ldots\ldots$$

This may be written in the forms

$$S=(a-b)+(c-d)+(e-f)+\ldots\ldots$$

and

$$S=a-(b-c)-(d-e)-(f-g)-$$

From the first of these forms

$$S>(a-b)$$

and from the second

$$S<a$$

Hence the series is convergent.

(At this point the student should solve ten to fifteen exercises illustrating the preceding theory.)

GEOMETRY.

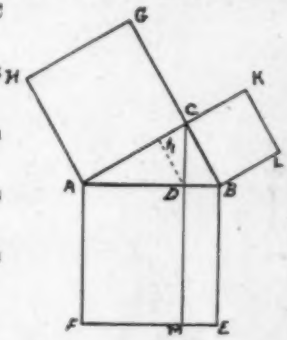
A High School Boy's Proof of the Pythagorean Theorem.

By Earl Davidson, '08, Kansas City, Mo.

Theorem: The square on the hypothenuse of a right triangle is equal to the sum of the squares on the two legs.

Given: Right triangle A B C, in which A B is the hypothenuse; and the squares, A B E F, A C G H, and B C K L, on the three sides.

To prove: Square A B E F=square A C G H+square B C K L.



Proof: Draw C D perpendicular to A B and produce C D to M in F E.

Draw h, altitude of triangle A C D, upon A C.

The measure of square A C G H equals A C × A C.

The measure of triangle A D C equals $\frac{A C \times h}{2}$

Then $\frac{\text{Square ACGH}}{\text{Triangle ACD}} = \frac{AC \times AC}{\frac{AC \times h}{2}} = \frac{2AC}{h}$

The measure of rectangle A D M F equals A F (or A B) × A D.

The measure of triangle A D C equals $\frac{A D \times D C}{2}$.

Then $\frac{\text{Rectangle ADMF}}{\text{Triangle ADC}} = \frac{AB \times AD}{\frac{AD \times DC}{2}} = \frac{2AB}{CD}$

Triangle A B C has the altitude C D drawn on the hypothenuse. Then triangle A B C is similar to triangle A C D. h and C D are homologous altitudes on the homologous sides A B and A C.

Then $\frac{2AC}{h} = \frac{2AB}{CD}$

Then $\frac{\text{Square ACGH}}{\text{Triangle ACD}} = \frac{\text{Rectangle ADMF}}{\text{Triangle ACD}}$

Therefore Square ACGH=Rectangle ADMF.

Since C D is the altitude of the right triangle A B C, upon the hypothenuse, triangle A C D is similar to C B D.

Since squares A C G H and B C K L are erected on the homologous sides A C and B C of the similar triangles, triangle A C D: triangle C B D = square A C G H : B C K L.

But since triangle A C D and triangle C B D have the same altitude, they are to each other as A D is to D B.

$\therefore$ square A C G H : square B C K L = A D : B D

But A D : B D = rectangle A D F M : rectangle D B M E.

$\therefore$ square A C G H : square B C K L = rectangle A D F M : rectangle D B M E.

Therefore square B C K L = rectangle B D M E, and square A C G H + square B C K L = rectangle A D F M + rectangle B D M E = square A B E F. *Q. E. D.*

[Math. Editor's Note.—This demonstration is original with Mr. Davidson according to his teacher. We take it from the *Nautilus.*]

A NOTE.

An Extension of the Pythagorean Theorem.

By Henry A. Converse, Ph.D.,

Davis and Elkins College, Elkins, W. Va.

Given three mutually perpendicular axes, OX, OY, and OZ. Let $OX = x$, $OY = y$, $OZ = z$. Then the sides of the triangle XYZ are respectively

$$\sqrt{x^2+y^2},\quad \sqrt{y^2+z^2},\quad \sqrt{z^2+x^2}.$$

Hence the area of XYZ is

$$S=\sqrt{\frac{\sqrt{x^2+y^2}+\sqrt{y^2+z^2}+\sqrt{z^2+x^2}}{2}\cdot\frac{\sqrt{x^2+y^2}+\sqrt{y^2+z^2}-\sqrt{z^2+x^2}}{2}\cdot\frac{\sqrt{x^2+y^2}-\sqrt{y^2+z^2}+\sqrt{z^2+x^2}}{2}\cdot\frac{-\sqrt{x^2+y^2}+\sqrt{y^2+z^2}+\sqrt{z^2+x^2}}{2}}$$

This reduces to

$$S=\sqrt{\left(\frac{xy}{2}\right)^2+\left(\frac{yz}{2}\right)^2+\left(\frac{zx}{2}\right)^2}$$

$$\text{or } S^2=\left(\frac{xy}{2}\right)^2+\left(\frac{yz}{2}\right)^2+\left(\frac{zx}{2}\right)^2$$

Hence the theorem:

Theorem.—In a tetrahedron one of whose trihedral angles is a tri-rectangular trihedral angle, the square of the area of the face opposite the tri-rectangular trihedral angle is equal to the sum of the squares of the areas of the other three faces.

PROBLEM DEPARTMENT.

IRA M. DELONG,

University of Colorado, Boulder, Colo.

Readers of the magazine are invited to send solutions of the problems in this department and also to propose problems in which they are interested. Problems and solutions will be duly credited to their authors. Address all communications to Ira M. DeLong, Boulder, Colo.

Algebra.

72. *Proposed by Wm. B. Borgers, A.B., Grand Rapids, Mich.*

At what time nearest after 6 o'clock are the hands of a watch in such positions that it would be possible for them to exchange places? And if they should exchange places, what time would it be then?

Solution by Eugene R. Smith, A.M., Montclair, N. J.

What is the first time after the Hth hour when the hands of a clock are interchangeable?

Let the hour hand move x minutes; then the minute hand will move $12x$ minutes. But if the hands had been interchanged, the minute hand would have passed through $5H + x$ spaces while the hour hand passed through $12x$ spaces $\therefore 5H + x = 12(12x)$ or $x = \frac{5H}{143}$. The minute hand has moved $\frac{60H}{143}$ minute spaces, and the time is $\frac{60H}{143}$ minutes past H o'clock. After interchange the time would be $5H\frac{144}{143}$ min. past 12. For this special case $H = 6$. Hence the time is $2\,\frac{74}{143}$ minutes past 6; after interchange it would be $30\,\frac{30}{143}$ minutes past 12.

[NOTE.—It is plain that the hands may be interchanged at any conjunction; moreover it is easy to show that another interchange may be made whenever the minute hand is $4\,\frac{8}{13}$ minute spaces ahead of the hour hand. It would seem therefore that it should be possible to state an equation whose roots give the instants within any hour interval when interchanges are possible.—ED.]

73. *Proposed by I. L. Winckler, Cleveland, Ohio.*

A man six feet high walks in a straight line at the rate of four miles an hour away from a street lamp, which is ten feet high. Find the rate at which the end of his shadow travels and also the rate at which the end of his shadow separates from himself.

Solution by W. T. Brewer, Quincy, Ill.

Since the lamp is four feet higher than the man's head, the end of his shadow is always $10 \div 4 = 2\frac{1}{2}$ times as far from the lamp as his head. Hence the end of his shadow travels at the rate of $2\frac{1}{2} \times 4 = 10$ miles an hour, and separates from himself at the rate of $10 - 4 = 6$ miles an hour.

Geometry.

74. *Proposed by H. E. Trefethen, Kent's Hill, Maine.*

Determine a point R such that lines drawn from it to the vertices of a triangle ABC make given angles with one another.

Solution by H. E. Trefethen, Kent's Hill, Me.

Let a, b, c, of which c is the largest, be the given angles to be subtended by the sides BC, AC, and AB respectively. There are three cases.

Case I. If $a + b + c = 360°$ and each angle is less than 180°, the point R is within the triangle ABC. On AB as chord describe a circle such that the segment toward C contains the angle c. On the opposite side of AB draw AD making the angle BAD the supplement of a and cutting the circle in D. Join DC cutting the circle again in R. R is the required point. For ARB $= c$ by construction. Also BRD $=$ BAD, being subtended by the same arc. Therefore BRC $= a$, being the supplement of BRD or its equal BAD.

Case II. If $c > 180°$, or if $c = a + b$ and $< 180°$, then R is outside the triangle. On chord AB describe a circle so that the segment *farther* from C contains the angle c (or, if c is given $> 180°$, contains $360° - c$). In the other segment make the angle BAD $= a$. Produce DC (or CD) until it meets the circle again in R. R is a required point. For ARB $= c$ and BRC $=$ BAD $= a$. Again on chord AB describe a circle so that the segment *toward* C contains c. Now if the point C falls inside this segment, there is also a point on this side of AB that satisfies the conditions of the problem. The construction and proof are the same as before. Thus in general there are two solutions when the angle $c <$ angle C and one solution when $c =$ or $>$ C. (More than two solutions are possible under some conditions.)

Case III. If $c = 180°$, then R is on AB. On BC construct a segment containing the angle a and cutting AB in R. R is obviously the required point.

75. *Proposed by C. R. Merrifield, Grand Island, Nebraska.*

If a wheel has 35 cogs and the distance between the mid-points of the cogs is 12 inches, find the radius of the wheel. [From Phillips and Fisher's Geometry.]

Solution by Ira P. Baldwin, Emporia, Kansas.

Circumference $= 12 \times 35 = 420$ inches $= 2\pi r$,

Hence $r = \frac{420}{2\pi} = \frac{420}{6.2832} = 66.84+$ inches.

[Phillips and Fisher evidently mean that the 12 inches is the circular distance. This is called "the circular pitch" of the gear wheel.—G. B. M. ZERR.]

Trigonometry.

71. *Proposed by E. L. Brown, M.A., Denver, Colo.*

From the vertices of a triangular field whose sides are a, b, c, the angles of elevation to the top of a perpendicular tower are observed to be α, β, γ, respectively. Required the height of the tower.

II. *Solution by H. E. Trefethen, Kent's Hill, Me.*

Let O be the foot of the tower and z its height. Take A for the origin. $AO = z\cot\alpha$, $BO = z\cot\beta$, $CO = z\cot\gamma$.

$$x^2+y^2=z^2\cot^2\alpha \quad \text{----------} \quad (1)$$

$$(x-c)^2+y^2=z^2\cot^2\beta \quad \text{----------} \quad (2)$$

$$(x-b\cos A)^2+(y-b\sin A)^2=z^2\cot^2\gamma \quad \text{--} \quad (3)$$

$$x=\left(\frac{\cot^2\alpha-\cot^2\beta}{2c}\right)z^2+\frac{c}{2} \quad \text{----} \quad \text{From (1) and (2).}$$

From (1) and (3) by substituting for x we have

$$y=\left(\frac{\cot^2\alpha-\cot^2\gamma}{2b\sin A}-\frac{\cot^2\alpha-\cot^2\beta}{2c\tan A}\right)z^2+\frac{b+c\cos A}{2\sin A}$$

In the last two equations writing for brevity m and n for the known coefficients of z^2 and p and q for the known terms, we have $x = mz^2 + p$ and $y = nz^2 + q$. Squaring $x^2 = m^2z^4 + 2mpz^2 + p^2$, $y^2 = n^2z^4 + 2nqz^2 + q^2$. Substituting these values for x^2 and y^2 in (1), collecting terms, and writing $2r$ for $\cot^2\alpha$ we have $(m^2+n^2)z^4 - (2r - 2mp - 2nq)z^2 + p^2 + q^2 = 0$. Solving

$$z=\sqrt{\frac{r-mp-nq\pm\sqrt{(r-mp-nq)^2-(p^2+q^2)(m^2+n^2)}}{m^2+n^2}}$$

76. *Proposed by J. M. Freeman, Boulder Creek, California.*

A point inside an equilateral triangle, side 50, subtends angles of 140°, 120°, 100° with the sides. Find the three lengths.

Solution by G. B. M. Zerr, Ph.D., Philadelphia, Pa.

Let ABC be the equilateral triangle, O the point within from which lines are drawn to the vertices.

Then AB = BC = CA = 50

Angle AOB = 120°, BOC = 140°, COA = 100°.

Let angle OAB = θ, angle OBA = ϕ, angle OCB = ψ

OA = x, OB = y, OC = z

Then $\theta + \phi = 60°$ ..... (1)

$60° - \phi + \psi = 40°$ or $\phi - \psi = 20°$ ..... (2)

$60° - \theta + 60° - \psi = 80°$ or $\theta + \psi = 40°$ ..... (3)

$50 : x = \sin 120° : \sin\phi$ or $x = 50\sin\phi / \sin 120°$ .... (4)

Similarly $y = 50\sin\theta / \sin 120°$ ..... (5)

$y = 50\sin\psi / \sin 140°$ ..... (6)

$z = 50\sin(60° - \theta) / \sin 100°$ ..... (7)

From (5) and (6) $\sin\theta / \sin\psi = \sin 120° / \sin 140°$ ..... (8)

(3) in (8) gives $\sin\theta / \sin(40° - \theta) = \sin 120° / \sin 140°$

$\therefore \cot\theta = \sin 140° / (\sin 120° \sin 40°) + \cot 40°$

$$\cot\theta=\frac{1}{\sin 120^\circ}+\cot 40' \quad \therefore\theta=23^\circ 4'\ 57.''46$$

$\therefore \phi = 36° 55' 2.54''$, $\psi = 16° 55' 2.54''$

$x = 50\sin(36° 55' 2.54'') / \sin 120° = 34.679$

$y = 50\sin(23° 4' 57.46'') / \sin 120° = 22.635$

$z = 50\sin(36° 55' 2.54'') / \sin 100° = 30.496$

Credits for Solutions Received.

Algebra 67. E. L. Brown, A. J. Wile.

Algebra 68. E. L. Brown, A. J. Wile, J. A. von Groos.

Algebra 72. H. E. Trefethen, W. B. Borgers, A. Grossman, A. J. Wile, E. L. Brown, W. T. Brewer, Wm. Campbell, M. L. Constable, J. L. Winckler.
Also one incorrect solution.

Algebra 73. J. Alexander Clarke, H. L. McAllister, R. P. Harker, G. B. M. Zerr, E. L. Brown, Grace MacLeod, Eugene R. Smith, F. J. Taylor, H. E. Trefethen, U. C. Barnett, W. T. Brewer, Paul P. B. Brooks, J. L. Winckler, W. H. Albrecht, H. C. Whitaker.

Geometry 69. E. L. Brown.

Geometry 70. E. L. Brown.

Geometry 74. Theo. L. Lindquist, F. J. Taylor, G. B. M. Zerr, Eugene R. Smith, E. L. Brown, H. E. Trefethen, H. C. Whitaker.

Geometry 75. F. J. Taylor, E. L. Brown, H. E. Trefethen, G. B. M. Zerr, Ira P. Baldwin, Grace MacLeod, U. C. Barnett.

Trigonometry 71. H. E. Trefethen, E. L. Brown.

Trigonometry 76. W. L. Malone, E. L. Brown, H. E. Trefethen, G. B. M. Zerr, Paul P. B. Brooks, Wm. Campbell, J. L. Winckler, H. C. Whitaker.

Total number of solutions, 57.

PROBLEMS FOR SOLUTION.

Algebra.

82. *Proposed by W. T. Brewer, Quincy, Ill.*

Solve $x^2 + y^2 + xy = 19 \ldots (1)$

$x^2 + z^2 + xz = 13 \ldots (2)$

$y^2 + z^2 + yz = 7 \ldots (3)$

Geometry.

83. *Proposed by I. L. Winckler, Cleveland, Ohio.*

Three circles, radii a, b, c, are drawn in a triangle, each circle touching the other two and two sides of the triangle. Find the sides of the triangle.—(*Mathematical Visitor*, 1878.)

84. *Proposed by E. L. Brown M.A., Denver, Colo.*

A circular field contains one acre. A horse is tied to a tree 20 feet from the field with a rope of sufficient length so that he can graze over one-fourth of the circular field. How long is the rope? E. L. B.

Trigonometry.

85. *Proposed by H. L. McAlister, Arkadelphia, Ark.*

A statue 10 ft. high standing on a column, subtends, at a point 100 ft. from the base of the column and in the same horizontal plane, the same angle as that subtended by a man 6 ft. high, standing at the foot of the column. Find the height of the column.—(Lyman & Goddard's Trigonometry.)

Miscellaneous.

86. *Proposed by Ira P. Baldwin, Emporia, Kans.*

3. A bather, standing on the bank of a circular pond three miles in circumference, wishes to reach the point directly opposite in the shortest time possible. If he can swim at the rate of 5 miles an hour and run at the rate of 6 miles an hour, what point should he swim toward?

A METRIC CARAT.

The weight of diamonds and other precious stones is still expressed the world over in carats. These units of weight are by no means uniform in different countries, but vary—according to a table of 22 States and cities cited by the Decimal Association of England—from 188.5mg. in Bologna to 213.5mg. in Turin, the average being about 205. An attempt, in which the Decimal Association is much interested, is being made to nuify these weight values by making a universal carat unit of 200 milligrams. The proposal is warmly endorsed by leading jewelers' associations in France, Germany, Belgium, Denmark, etc. The new unit is to be called the metric carat, to distinguish it from others. Its universal adoption would greatly facilitate international trade in precious stones.

R. P. W.

THE SCHOOL AS THE INSTRUMENT OF THE STATE.

Mail-Study Departure at the University of Wisconsin.

Last summer, late in the session of the Wisconsin Legislature, a bill was passed establishing a correspondence school as a department of the State University at Madison. This establishes the crowning feature of the admirable educational system of that state.

Provision now exists in the state system of education whereby not only the youth of the commonwealth from the kindergarten to the universities have educational opportunity, but the large group of unclassified adults of all ages and all degrees of advancement is now also guaranteed a responsible standardized system of instruction which may be pursued at home through the mails. This work is being made largely practical, and to relate effectively in one way or another to the problems of life confronted by such an adult class of students. The artisan or the clerk may receive elementary and technical training; the professional man may utilize the new department for keeping abreast of the additions research is constantly making in every field of knowledge; and the teacher may earn a college degree, "learning while earning."

Correspondence students who are residents of Wisconsin have, beside, exceptional co-operating assistance from the state library system.

This establishes a new precedent for State Universities in extending educational services to every productive interest in the state similar to those so long and so effectively rendered by the agricultural colleges alone. This is one aspect of President Van Hise's interesting policy of "making the university the instrument of the state."

DEPARTMENT OF SCIENCE QUESTIONS.

FRANKLIN T. JONES,
University School, Cleveland, O.

This department is designed to serve as a medium for the exchange of ideas on questions and questioning in the sciences. Questions will be printed from various sources—college entrance examinations, text-books, etc., etc. Comment is invited. Suggestions and criticisms as to character, adaptability, and usefulness are desired. Readers of this journal are invited to propose questions and problems which will be of general interest, or of a type which will be useful in the class room. It is not expected that questions which will not be useful to pupils will be frequently printed.

Since the majority of the questions will be of a comparatively simple character, solutions and answers will not be published unless specifically asked for. Teaching suggestions are wanted.

Address all communications to the editor of the department.

Please send copies of examination papers in the sciences to the editor of the department.

In May the papers of the College Entrance Examination Board appeared. These papers have aroused a great deal of comment, and some of the criticisms will be made public.

It is assumed that in the giving of an examination the idea is (1) to test both attainment and memory, but primarily the former; (2) to avoid technicalities and at the same time to frame questions that require direct and moderately brief answers; (3) to make the questions straightforward, on fundamental points, and reasonably severe; (4) to give ample but not too much time to write the answers; (5) to be fair to both pupil and teacher.

No severe criticism of the physics and geography papers were received. so it can be supposed that these questions are reasonably satisfactory. One comment on the botany paper applies equally to all: "If these questions are fundamental why omit any of them; I would prefer nine questions only that a pupil should be able to answer. Perhaps the latitude is given in view of the diversity in the courses of study in various schools." "The questions are not too difficult, nor do they presuppose more work than the pupil can cover."

Questions on chemistry were less satisfactory. The following answer is an excellent summary of the opinions expressed:

"(a) The questions are not of the right kind. They are tests of memory based on a knowledge of the text. They are not technical enough nor general enough. (b) Considering the time specified—2 hours —the paper is too easy. (c) They do not necessitate a knowledge of principles. (d) It is my impression that, as chemistry is commonly taught, the paper is pretty fair to both teacher and pupil. The choice of questions ought to make it very easy—one that all the students could pass without ever having done very hard work on the subject. * * *

The paper does not seem to require a knowledge of the periodic arrangement of the elements.

"There is not enough equation work. It does not seem to lay stress on the important transformations with which every student must be perfectly familiar if he hopes to be able to think in chemistry. Oxidation reactions seem to be entirely lacking. Nothing is said about the stage of oxidation and how to obtain a different stage."

Careful comparison of the papers of the College Board, Sheffield Scientific School, and Princeton as published in the November number will be very interesting and profitable. Are the last two sets of questions open to similar criticisms?

Is chemistry receiving the attention it should in secondary schools?

Is there any reason why chemistry should not have as prominent a place in the curriculum as physics?

Is not chemistry better suited to high school study than physics?

Harvard University. 1907.

Botany

(One Hour)

All notebooks must be handed in at the laboratory examination, and must be claimed when it is over.

Use drawings, when possible, to make your answers clear.

Omit one question.

1. Describe the principal parts of a complete seed, and give an account of the chief changes which each of these parts goes through in germination.
2. Describe any two kinds of roots (for example, ærial, parasitic, saprophytic, etc.), and give a short account of the manner in which they perform their function.
3. Describe any plant-movement which you have observed.
4. Where do buds occur? What is their structure? What office do they have in temperate climates?
5. Describe the work which is done by green foliage.
6. Describe the structure of any flower in which close fertilization is impossible.
7. Give the principal characters of any Order (Family) of flowering plants.

Zoology

(One Hour)

All notebooks must be handed in at the laboratory examination, and must be claimed when it is over.

Answer eight of the following questions.

1. Describe the eyes of a grasshopper, and show in a sketch where on the body they are located.

2. Describe the feeding habits of a caterpillar.
3. Upon what kinds of substance do beetles feed? How do their mouth parts differ from those of bugs?
4. How many wings has a fly, and to what body-segments are they attached.
5. Can starfishes see? What evidence can you give in support of your opinion?
6. Describe the mouth parts and the feeding habits of Nereis.
7. How do earthworms move?
8. Describe the spinning organs and the webs of spiders.
9. Describe the appendages of Lithobius.
10. How does a crayfish breathe?
11. Where may Daphnia be found, and how may it be distinguished from other animals often associated with it?
12. Where do slugs live, and how do they move?
13. How do the two valves of the clam-shell differ?
14. How does Hydra reproduce?
15. How does Paramecium secure its food, and of what does this consist?
16. How and where do frogs pass the winter?
17. How many appendages has a fish? what corresponding parts has a salamander?
18. Where do lizards live, and how do they reproduce?
19. How do the feathers of a young sparrow differ from those of an adult?

Sheffield Scientific School.

BOTANY.

[The candidate may take the examination in Botany, Physics, or Chemistry.]

1. What is an herbaceous plant? What is the difference between annual and biennial herbs? Give an example of each and describe, in the case of the biennial, the structures in which food is stored.

2. What is a compound leaf? Define, with characteristic examples, the two principal types of compound leaves. How may a compound leaf be distinguished from a flat branch with two longitudinal ranks of simple leaves?

3. Describe the general features of the root with special reference to its work of absorption. Show how osmosis plays an important part in this process.

4. Describe the appearance and structure of the bark in an old tree. What functions does it perform? Why does a tree die when a complete ring of bark is removed?

5. What are parasitic plants? How does a green parasite differ in its nutritive processes from one which lacks chlorophyll? Give an example of each kind.

6. Describe two ways in which plants may prevent self pollination and secure cross pollination. Illustrate by examples.

SOUTH DAKOTA ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

The first annual meeting of the South Dakota Association of Science and Mathematics Teachers was held in Sioux Falls, October 18th and 19th. The following papers were presented: "High School Geometry," Professor G. H. Scott, Yankton College; "Interest in Introductory Science Aroused and Sustained," F. Ethel Smith, Huron High School; Discussion—"The New Movement Among Physics Teachers," R. C. Shellenbarger, Supt. of the Yankton College, Mr. F. Schumacher of the Sioux Falls High Schools; "Some Facts and Theories from the Field of Biology," Prof. C. P. Lornmen of the University of South Dakota.

The following resolution was passed regarding the Physics unit: *Resolved*, That the unit in Physics should be 135 hrs. instead of 150 hrs.

The attitude of the association toward the new movement among Physics teachers was expressed by the following resolution: *Resolved*, That less quantitative work should be required in the course in High School Physics, and more qualitative work be introduced, making Physics in the secondary schools a natural rather than a mathematical science.

"The purpose of the association," as defined in the constitution, "shall be to promote the better teaching of Science and Mathematics in the schools of South Dakota; to obtain better correlation of these subjects to each other and to the other subjects of the curriculum; to bring the colleges and secondary schools into closer relations in the teaching of these subjects and to establish cordial and helpful relations among teachers of these branches."

Although the attendance at the meetings was small, the papers were exceedingly helpful and the discussions full of interest. All who attended expressed themselves as abundantly repaid.

NEW JERSEY SCIENCE TEACHERS ASSOCIATION.

The spring meeting of the New Jersey Science Teachers' Association was held at Princeton University on May 11th. The following officers were elected: President, W. Wiener, Newark High School; Vice-President, Gilbert H. Trafton, Passaic High School; Sec.-Treasurer, E. A. Buck, Trenton High School; Executive Committee, the officers, the ex-president, A. T. Seymour of Orange High School, Caroline S. Romer Newark High School, G. B. Roorbach, Peddie Institute, Hightstown, H. M. Campbell, Long Branch High School.

Mr. Wiener of Newark High School read a paper on "Opportune Methods in Science Teaching." In this paper were discussed some of the fundamental principles that underlie all successful science teaching. The need of an enthusiastic, clean-cut treatment of the topics in class discussion by the teacher as a means of arousing a corresponding interest and enthusiasm on the part of the pupil was emphasized. The amount of home work assigned should not be long but short and to the

point, nor should the pupil be restricted to one text-book in this work but should be allowed the choice of several.

Mr. Ray Whitbeck of the Trenton Normal School read a paper on "Secondary School Geography: What Shall It Be?" Statistics were quoted showing that all the sciences had relatively lost ground in the last few years. The remedy for this in Physical Geography lies in humanizing the subject, to teach not Physiography but Physical Geography. Recent text-books go too much into detail in the classification of the various land forms and have too little to say regarding the influence these have exerted on human life. It is questionable whether much of the laboratory work done in the subject is of educational merit; and indeed the subject does not lend itself as readily to the laboratory method as do the other sciences and does not require so much of this kind of work for its highest development. In order to redeem itself Physical Geography must deal more directly with the influence of physical forms and features upon the inhabitants of these localities.

Prof. E. P. Adams of Princeton Univ. gave a lecture on "Radio-activity." A brief summary was given of the researches that had been carried out along this line; and the chief substances found to possess this power were enumerated. The two chief tests of this power are its effects on a photographic plate and the rendering of air a conductor of electricity. Both elements and compounds are found to possess these powers, but radium in largest quantities. There is a remarkable relation between the radio-active elements, uranium, actinium, radium, and polonium, by which each one changes gradually to the one following when allowed to stand for a certain length of time, and the probabilities are that the last element in this series may be lead. When radium stands, helium is gradually formed, this apparently being an emanation from the radium as it is changing to polonium. But there is nothing in these experiments which suggests that other elements may change from one to the other.

Prof. W. B. Scott of Princeton Univ. gave a lecture on "South Africa —Its Geography and Geology." The surface form is simple, consisting of dry dissected tablelands wth some coastal plains at the border. The interior is dry and arid and has much the appearance of the plateaus and plants of the dry regions of Montana, but the plants are of entirely different families. Granite is one of the most common rocks, which, in that climate, weathers into fantastic shapes, suggesting some features of Yosemite Valley. Evidences of enormous glacial action in very ancient ages is of a most convincing kind. In some places the till is 1,000 feet deep, thus giving some evidence of the tremendous scale in which glacial action took place. Besides the till, glacial boulders and striae both on bed rock and loose pebbles were very conspicuous. Strange to say, the glacier moved not toward the equator, but towards the pole. The great falls of South Africa present a maze of complications owing to the irregular jointing of the rocks, making a series of falls spreading out over many hundreds of feet and then uniting into a tortuous narrow gorge below the falls.

PERSONAL NOTES.

Miss Laetitia Goslin has just become instructor in Mathematics at Waterloo, Iowa.

Arthur Durand of Pomona, Cal., accepts the principalship of Bonita, Cal., High School.

Joseph Bikle has been appointed Associate in Mathematics at Bradley Polytechnic Institute.

Louis C. Plant has been appointed associate professor in the University of Montana at Missoula.

A. F. Leonard of East Weymouth, Mass., goes to Maine as Principal of the High School at Camden.

Miss Grace Barker, S.B., Chicago, 1907, is instructor in Science in the University Schools for Girls, Chicago.

Miss Ethel Terry, A.B., Chicago, 1907, is teaching Physics and Chemistry in Kenwood Institute, Chicago.

W. A. Fiske of Richmond, Ind., goes to Occidental College, Los Angeles, Cal., as professor of physics and chemistry.

Mr. S. W. Ward is just entering upon his work as instructor in Physics and Chemistry in the high school at Kenosha, Wis.

Miss Etoile B. Simons, Ph.D. (Chicago), has been appointed instructor in Botany in the Manual Training High School, Indianapolis.

Professor Wilbur Olin Atwater, since 1873 professor of chemistry at Wesleyan University, died, on September 22, aged fifty-three years.

Mr. John E. Stout is the new principal of the township high school at LaGrange, Ill., in the place of E. B. Cole, who recently resigned.

Bertram G. Smith of the University of Michigan has been appointed to succeed Dr. Reese as professor of zoölogy in Syracuse University.

Samuel G Barton, Ph.D. (Pennsylvania), has been appointed to the chair of mathematics in the Clarkson School of Technology, Potsdam, N. Y.

A. S. Pearse, Instructor in Biology, Lake High School, Chicago, has just received an appointment as a teaching fellow at Harvard University.

Dr. Lauder W. Jones, for several years instructor in chemistry in the University of Chicago, has recently been appointed to the head of the department of chemistry in the University of Cincinnati.

Dr. David Eugene Smith, Professor of Mathematics in Teachers College, Columbia University, on July 26, 1907, sailed from San Francisco on sabbatical leave of absence to take a trip around the world and to visit the old centers of mathematical interest. While in Kioto and Tokio recently Dr. Smith secured some five hundred old Japanese books and manuscripts, many over 200 years old, beside a number of rare instruments. In Kioto he gave three addresses, visited the schools and attended numerous functions. In Tokio he visited the University and met several of the leading Japanese mathematicians. On September 26, Dr. Smith sailed from Kobe for Tangku, China. He purposes to arrive in Rangoon early in October, visiting Calcutta, Bombay, and Colombo during November, December, and January. In February his

plans will lead him to Cairo and later to Constantinople. It is possible he will attend the Congress of Mathematicians which takes place in Rome in April. In all these cities he will visit the schools and colleges to study the progress of mathematics and to look into the educational conditions. Visitors and students at Teachers College in September, 1908, will have the rare opportunity of seeing the most unique collection of mathematical curios and rare books ever gotten together. Those taking the courses offered in the department of mathematics will have an opportunity of hearing from Dr. Smith many details of his trip, which promises to be of great interest and value to his students.

ARTICLES IN CURRENT MAGAZINES.

The *American Naturalist* for September contains: "The Structure of Cilia, especially in Gastropoda," by Leonard W. Williams; "The Poison Glands of *Notorus* and *Schilbeodes*," by Hugh D. Reed; "The Structure of the Silk Glands of *Apanteles Glomeratus*," by Robert Matheson and A. G. Ruggles.

Astrophysical Journal, November: "Physical Motion of Meteor Trains," C. C. Trowbridge; "On the Doppler Effect in the Spectrum of Hydrogen and of Mercury." G. F. Hall.

Biological Bulletin for September: "Studies on the Relation between Amitosis and Mitosis," C. M. Child; "Can Sea Water Maintain the Beat of the Heart of Fresh Water Animals?" Theo. C. Burnett; "The Fertilization of Amoeba Proteus," Gray N. Calkins.

Economic Geology for September-October: "The Clays of North Dakota." C. H. Clapp; "The Cuyuna Iron Range." Newton H. Winchell; "Possible Oblique Faulting in Alaska," Lawrence Martin.

Garden Magazine for November: "How to Keep House Plants Healthy." P. T. Barnes; "A Guide to the Chrysanthemum Shows," I. L. Powell; "The Best Hedge Plants South and North." Prosper J. Berckmans; "Grafting Your Own Conifers." John Dunbar; "The Plain Truth about Chestnut Culture." J. W. Kerr; "Succulents Other than Cacti." W. Clarke.

Journal of Geology for September-October: "The Pre-Richmond Unconformity in the Mississippi Valley." Stuart Welles; "On the Origin and Definition of the Geologic Term Laramie," by A. C. Veatch; "The Characteristics of Various Types of Conglomerates," George R. Mansfield; "Dome Structures in Conglomerates," Ralph Arnold; "Pre-Wisconsin Drift in the Finger Lake Region of New York," Frank Carney.

Open Court for November: "The Evolution of Climate," by Lawrence H. Daingerfield of the Weather Bureau.

Ores and Metals for Sept. 20: "Tin Mining in the Federated Malay States," by E. Nightingale; for Oct. 5: "Electricity in the Joplin Mining District;" for Oct. 20: "Zinc, Its Minerals and Its Metallurgy," by Evans W. Buskett.

Photo Era for November: "Long-Focus vs. Short-Focus Lens." C. H. Claudy; "The Shutter Problem." Harry Edwards; "Advantage of a Small Camera," W. F. Zierath; "Moving Objects, a Note for Hand Camera Workers," A. G. Workman.

Physical Review for October: "The Influence of Molecular Structure upon the Internal Friction of Certain Isomeric Ether Gases." Frederick M. Pedersen; "The Distribution of Energy Emitted by a Righi Vibrator." C. R. Fountain and F. C. Blake; "A Study of the Reversible Pendulum. Part I. Theoretical Considerations." John C. Shedd and James A. Birchby; "Measurement of Electrolytic Resistance," W. S. Franklin and L. A. Freudenberger.

Popular Astronomy for November: "The Mills Expeditions from Lick Observatory," W. W. Payne; "The Age of the Earth's Consolidation," J. J. Lee.

Popular Science Monthly, October: "A Trip Around Iceland," L. P. Gratacap; "The Sacrifice of the Eyes of School Children." Professor Walter D. Scott; "Notes on the Development of Telephone Service." Fred DeLand; "Linne and the Love for Nature." Edward K. Putnam; "Early Movements in the United States for a National Observatory," Charles Oscar Paullin;

"What Pragmatism is Like," Giovanni Papini; "The Problem of Age, Growth and Death." Professor Charles S. Minot. For November: "The Scope and Importance to the State of the Science of National Eugenics," Professor Karl Pearson; "A Trip Around Iceland." L. P. Gratacap; "Notes on the Development of Telephone Service," Fred DeLand; "The Institute of France and Some Learned Societies of Paris," Dr. Edward F. Williams; "Fertility and Genius," Charles Kassel; "The Problem of Age, Growth and Death." Professor Charles S. Minot.

Review of Reviews for October: "The Japanese Case in Korea," Adachi Kinnesuke; "McKinley's Memorials in Sculpture," E. H. Brush; "The Improvement of Our Inland Waterways," W. F. Saunders; "A Yankee Engineer on Five Continents," D. F. St. Clair.

Scientific American Supplement for Oct. 5: "The Arsenic Industry of Cornwall;" "The Colt of the Cactus," by S. Leonard Bastus. Oct. 12: Roman Cities of Northern Africa," 11 illustrations. Oct. 26: "The Temperature of the Sun," by A. Millochan assistant astronomer at the Mendon Observatory. Nov. 2: "Deformation and Variation in Sea Level," H. W. Pearson.

Western Electrician for Nov. 2: "Some Features of the Rebuilding of Chicago's Street Railways," illustrated.

BOOKS RECEIVED.

Geometric Exercises for Algebraic Solution. Second-Year Mathematics for Secondary Schools. By George W. Myers, Professor of the Teaching of Mathematics and Astronomy, College of Education of the University of Chicago. Pp. ix + 71. University of Chicago Press.

First-Year Mathematics for Secondary Schools. by George W. Myers. 198 pages. 12mo, cloth. Net, $1.00; postpaid, $1.09. University of Chicago Press.

A History of the Illinois State Museum of Natural History.

Bulletin 34. New York State Science Teachers Association, proceedings of the eleventh annual conference.

When Men Grew Tall. The story of Andrew Jackson. Alfred Henry Lewis. 1907. D. Appleton & Co., New York. Price, $2.00.

Nature and Development of Plants, by Carlton C. Curtis, Instructor in Botany in Columbia University. Pp. 471. Henry Holt & Co., New York. 1907. Price, $2.50 net.

Progress in Agricultural Education, 1906, by Dick J. Crosby. Reprint from Annual Report of the Office of Experimental Stations for 1906. U. S. Dept. of Agriculture.

Plant Physiology and Ecology, by F. E. Clements. Pp. xv + 315. Henry Holt & Co., New York. 1907. Price, $2.00.

Computation and Mensuration, by P. A. Lambert. The Macmillan Company. New York. 1907. Pp. 92.

The principles of Agriculture, a text-book for schools and rural societies, by L. H. Bailey. 12th edition. The Macmillan Company, New York. 1907. Pp. 300.

Higher Education, Bulletin 3.

Professional Examination Papers for the Academic Year, 1906-07. N. Y. State Education Dept., Albany, N. Y. Price, 25 cents.

Elements of Biology, a practical text-book correlating botany, zoölogy, and human physiology, by G. W. Hunter, Instructor in Biology, Detroit Clinton High School. New York. Pp. 443. American Book Co. 1907.

BOOK REVIEWS.

ELEMENTARY TURNING, by Frank Henry Selden, Instructor in Woodworking, University of Chicago, High School. Rand, McNally & Co. pp. 197, 12.5 cms × 17.5 cms. Price, $1.00.

This is the second of a series of texts on wood working being prepared by Mr. Selden. Instructors in manual training have been looking for a book of this character for years. With this text in hand, almost any one ought to be able to quickly become familiar with the turning of wood in its most intricate forms. The book consists of three parts. Part one, of 103 pages, is devoted to thirty elementary lessons in the art; Part two, of 62 pages, to thirty-two supplemental exercises, and Part three, of 30 pages, to tools and fittings. There are 298 illustrations, 274 of which are half tones, showing how the work is actually done. These illustrations are very largely taken while the lathe was being used, thus the pupil will have no difficulty in determining just how his work and tools should be held. The exercises are clearly and concisely written. The book embodies the skill and experience of a master in this art. S.

New Elementary Arithmetic. By George Wentworth. 12mo, cloth; 232 pages; illustrated. List price, 35 cents; mailing price, 45 cents. Ginn & Company, Publishers.

No one subject serves so well to develop in the young accuracy, speed, and logical habits of mind as does the early training in arithmetic.

Wentworth's New Arithmetic aims to satisfy these conditions. It approaches each subject logically by means of a series of questions, the answers of which are easy to discover if not already known. Its exercises and problems, of which there are a great number, have been selected and graded with special care; and its wording is simple and readily understood.

The greatest stress is laid upon repetition, which, enlivened by interesting variations, tends to produce speed and accuracy of calculation. Easy fractions and the more common measures are introduced early, and will be found of use in stimulating the interest of the pupils in their work.

The book is intended for pupils in the second, third, and fourth grades, the text being arranged in five chapters, each of which should constitute the work for a half year.

It includes, in addition, enough material in denominate numbers, decimal fractions, bills and receipts to assist greatly the work in the fifth and sixth grades. S.

The Book of Vegetables and Garden Herbs, by Allen French. The Macmillan Company, New York. Price, $1.75 net.

In an illustrated volume of over 300 pages the author gives directions for the planting and cultivation of garden vegetables, including not only those usually found in home and market gardens, but also many kinds little known in America, but cultivated in France and England. The

soil required for the best crops of various plants is discussed, and an extensive use of various chemical fertilizers recommended. Detailed directions are given for planting; the depth, distance apart, and amount of thinning required being separately treated for each variety. Useful hints are also offered as to how the crop should be cultivated, protected from disease, and harvested.

A glance through the book leaves the reader with the impression that more of the information contained within its covers has come from other books on horticulture and from the bulletins of experiment stations than from the authors' personal experience. It is, however, likely to prove most useful to the amateur gardener, and can be recommended for the use of teachers who are attempting school gardens without having had a thorough training in horticulture. It is made more suitable for use in the school by giving the scientific as well as the horticultural names of the vegetables, but the use of chemical fertilizers recommended is too elaborate and extensive for the ordinary school garden.

The alphabetical arrangement of the plants under their common names makes it very convenient as a book of reference.

GEO. D. FULLER.

The Common Bacterial Infections of the Digestive Tract, by C. A. Herter. Pp. iv + 360. The Macmillan Company, New York. $1.50.

This most interestingly written volume by Dr. Herter of the New York City Hospital has presented the results of recent work upon the bacteria of the human digestive tract. Whether bacteria are necessary to ordinary digestive processes as claimed by Pasteur, Nuttall, Thierfelder, and others, the author says cannot be finally stated at this time. The fact that bacteria are absent from the digestive tracts of some Arctic animals certainly proves that some animals may live without this sort of bacterial assistance. The author is of the opinion that the bacteria so commonly found in the human digestive tract may be of assistance indirectly at least by preventing the growth of forms that would be postively harmful if allowed to flourish. He shows further that the number of bacteria inhabiting bottle-fed children exceeds the number present in those normally fed, due to the much larger number of bacteria present in cow's milk. The number merely does not indicate especial dangers, but "those specimens of milk which contain the largest numbers are also most likely to contain the forms which aside from inciters of typhoid fever and dysentery, are most injurious."

An interesting chapter is the one upon "The Bacterial Conditions in the Digestive Tract during Childhood, Adolescence, Adult Life, and Senescence." Extensive variations between these periods exist, and although some light is had as to the causes, further work alone can give positive results. It is clear that "the outset of senility may be distinctly accelerated through the development of intestinal infections in which the putrefactive anaerobes are prominently represented." Other chapters of especial importance relate to "Characters of the Bacterial Flora of Carnivorous and Herbivorous Animals," and various

phases of intestinal putrefaction. A short chapter on "Sociological Considerations," giving from a physician's point of view some wholesome and timely suggestions, is full of good sense and appropriate advice.

Teachers of bacteriology and of human physiology will find this book of great help in their work. O. W. C.

Laboratory Equipment for Physical Geography. Central Scientific Company, Chicago.

Among the new publications of interest to Science teachers, especially those teaching Physical Geography, will be found an 85 page Physical Geography Catalogue published by the Central Scientific Company of Chicago, Ill. This catalogue is unique, being, as far as we know, the only exclusively Physical Geography Catalog, and is on par with the progressive ideas noted in all the publications of this Company.

The authors in this catalogue have used the different Physical Geography Text Books as a basis, and there will be found in this catalogue any apparatus called for by the various text books arranged in alphabetical order. Some original ideas are noted in addition to new and up-to-date Meteorological Instruments. Twelve Relief Models are catalogued, nine of which are the models by Prof. Harry Keeler, the reputation of which are already so well established. Several pages are devoted entirely to descriptions and lists of maps of various kinds, especially arranged in sets to suit individual needs of the teacher.

A very complete series of topographical maps, sea coast charts and miscellaneous maps is also listed in this section. Two pages are devoted to a list of various globes, plain, medium and full mounted, as well as slated globes of different sizes and features. Three pages are devoted to lists of lantern slides and accessories for lantern slide making. Two pages are devoted to lists of minerals and soils. We understand many new numbers are to be noted in this catalogue.

The last four pages are devoted to Special Lists of apparatus. These lists are based on the various manuals in use. Darling's Manual published by Atkinson, Mentzer & Grover, Darling's Loose-Leaf Manual used in the Chicago High Schools, and the apparatus needed to accompany the new Manual by Blount, Everly & Walton, at present in the hands of the printer. These lists have received the approval of several prominent Physical Geography teachers and promise to be a very efficient aid to teachers in equipping their laboratory.

Three pieces of furniture are listed. These have not as yet appeared in other catalogues of this company and will be noted with interest by many. The Lantern Table, Mineral Cabinet and New Physical Geography Table are unique in design and will surely appeal to all in need of convenient and efficient furniture.

Being a very complete list of needs of a Physical Geography Laboratory, teachers desiring such supplies may secure valuable information therefrom. The catalogue will be sent free of charge upon application to the Central Scientific Company, 14 Michigan St., Chicago, Ill.

C. M. T.

Laboratory Exercises in General Zoölogy, by Glenn W. Herrick. Pp. 110. American Book Company, 1907.

This book is intended as a guide for the laboratory exercises that should accompany recitation work with the text-book of General Zoölogy, written by the same author and recently published. Not only is it well calculated as a supplement to the text mentioned, but is adapted as a basis for an elementary laboratory course where other text-books are used. The material is well selected and is presented in a logical manner, and there are many valuable suggestions for both teachers and pupils.

The first part of the manual deals with "Preliminary Exercises," explaining the use of the compound microscope in a clear and concise way and also giving some practical lessons on "Functions of Organs," "Classification," "Plant Cells," and "Animal Cells." These preliminary exercises will thus acquaint the pupil with some general biological principles, that will give more meaning to the further study called for by the book.

There are nineteen lessons on types of animals. These types are, for the most part, the same ones made prominent in the companion text-book. The order of consideration is the one beginning with the simplest forms and proceeding to the most complex. The exercises are of such a nature that a different arrangement of lessons can be used if the teacher so desires. The author has striven with much success to make the order of presentation a flexible one. Beginning each exercise, there is given, in small type, a list of materials needed to do the work called for and also suggestions for obtaining and preparing the specimens required. Then, in larger type, are given the directions for procedure in the study of the type. The method here is chiefly by asking questions, which the pupil is expected to answer. To do this work, the student must in many cases not only examine the specimens in hand but must perform experiments and study the living forms in the field. The book is, therefore, not of the common type of laboratory manual, chiefly for morphological work, but is designed for physiological and ecological studies, and it is a commendable feature that details of structure are not often pointed out to be examined, but the pupil has his attention called to only those parts that he must be familiar with in order to understand the important activities of the living animal.

The some sixteen pages of appendix give valuable facts concerning equipment for work, preparation of material, and the making of and caring for aquaria and cages. A list of forty good reference books is given, also the names and addresses of dealers in laboratory supplies.

T. L. HANKINSON.

A Manual of Biological Projection and Anesthesia of Animals, by A. H. Cole, Chicago Normal School. 200 pages, 28 figures. Published by the Neeves Stationery Company, Chicago, 1907.

The extended uses to which various kinds of projection apparatus have been put in recent years will receive further stimulus through a book designed to show the results of the author's work on projection

and anesthesia of animals. Many teachers of long experience in using the projection lantern will find new points of value in the preliminary chapters relative to the question of the kinds of illumination that are available, and the kinds of apparatus that may be adjusted to these lights. Most of us have found that one kind of light is all that we care to master and adjust to our apparatus, and agree with Professor Cole that the electric arc is the best of all available artificial lights. The author, however, concludes that sunlight is for all classes of projection superior to all forms of artificial light, and has additional advantages in being cheaper and easier to use. The numerous demonstrations of the use of sunlight will doubtless be of interest to every reader. Ample directions are given for adjusting the Porte-Lumiere, for darkening rooms, and for selecting and adjusting magnifying and projection lenses.

Perhaps the most valuable part of the book is that dealing with anesthesia of animals and plants—mostly animals. Chloretone has been used in various ways as an anesthetic and has been found superior to all other agents. It has been used in the projection and study of animals ranging from the amœba to the frog with highly commendatory and truly wonderful results. Different types of cells for containing plants and animals, and the manipulation of these plants and animals are treated in a most helpful way.

There is doubtless a danger that devices for excellent projection may lead some teacher to attempt to eliminate indispensable first-hand contact with the actual materials of laboratory work. First-class pictures of things are next to the things in value. In many cases minute objects may be magnified and projected so as to be much more easily studied than may be done by individual students under separate high power microscopes. Many life phenomena may be more easily observed in this way. But many of the values of good laboratory work will be very hard to secure if the habit of having an entire class study from a screen regularly should become general. Putting aside the question of whether projection should assume a much larger role in biology teaching it is eminently true that Mr. Cole's book is so full of valuable suggestions as to make it most welcome to all who teach biological subjects. O. W. C.

Text-book in General Zoölogy, by Glenn W. Herrick, Professor of Biology in the Mississippi Agricultural and Mechanical College. 386 pages, 237 figures. Published by the American Book Company.

Mr. Herrick's book is a very desirable addition to the number of good texts for elementary zoölogy courses which have recently appeared. It will be especially welcomed by those teachers who wish to give a well-proportioned course in zoölogy, dealing chiefly with the important facts of the Animal Kingdom as a whole and not giving special stress to certain groups or to particular lines of study like ecology, physiology, and morphology. Each branch of the Animal Kingdom is discussed, and then following this systematic consideration are some chapters on general topics; life processes, geographical dis-

tribution, struggle for existence, animals of the past, and also a short chapter on the history of zoölogy.

The branches of the Animal Kingdom are discussed in the order beginning with the Protozoa and ending with the Chordata. The classification followed is similar to that used in Parker and Haswell's text-book. Some important differences from this are that the Arthropoda is elevated to a position just below the Chordata, highest in rank among the invertebrates and the Annulata is subordinated to a place below the Echinodermata. The treatment of each branch is similar to the excellent method of Parker and Haswell in that a type species is first discussed somewhat in detail, and then follows a general consideration of the group as a whole. The types are well chosen. The more important ones are Amœba, Grantia, earthworm, starfish, river mussel, crayfish, Carolina locust, perch, green frog, six-lined lizard, English sparrow, and rabbit. At the end of the discussion of each branch or important class is a table showing its more important subdivisions and their subdivisions and so on down to orders. Type species of each of the orders are given. Important facts concerning the habits, ecology, and economic relations are included with those on morphology and physiology in the consideration of each group. More such information, it seems, might have been presented in many cases, that would have furthered the purpose of the text-book without much increasing its size. This would certainly have added to its attractiveness and value and to its desirability as a text for the many zoölogy teachers who are striving at the present time to teach the natural history side of the subject.

Much assurance that the material in Mr. Herrick's book is of high quality comes from the fact that some twelve of the best specialists and scholars in the separate fields of zoölogical knowledge have read and criticised portions of its manuscript. The names of these persons are given in the introduction of this book. It is also very favorable to the character of the work that it was written by a teacher of ten years' experience in teaching the subject matter of the book to the grade of pupils for which it is intended.

The author states his purpose in writing the text as follows: "The aim has been to create an interest in nature, beget an acquaintance with the lives, habits, and activities of animals, train the powers of observation, quicken the judgment, widen the horizon of environment, augment the capabilities for independent thinking, and inculcate an unswerving regard for the truth." This, it seems to the reviewer, is a good statement of the ideal purpose of a high school biology course, and in the hands of a good teacher, used as a basis for recitation work and along with well-planned laboratory and field work, this book should be very instrumental in accomplishing this end.

T. L. HANKINSON.

Laboratory and Field Manual of Botany, by Joseph Y. Bergen, A.M., and Bradley M. Davis, Ph.D. Pp. viii + 257. Ginn & Co. 1907.

This book consists of three parts. Part I contains "studies on the

cross anatomy and histology of seed plants," with a "set of experiments to illustrate some of the main principles of physiology." Part II deals with "type studies of spore plants and outlines the evolution and classification of the plant kingdom." Part III offers a series of laboratory and field studies presenting "at least an outline for the treatment of ecology as a scientific subject." Parts I and III were written by Mr. Bergen and Part II by Mr. Davis.

The book is said by the authors to offer much more than a year's work in botany. This is done, they say, to give a chance for selection to teachers of botany. They seek to assist the teacher to a wise choice by printing the studies in two sizes of type and also by indicating the most important studies by double stars. This makes a cumbersome set of studies and unfortunately the pupils must buy the entire work even though the teacher uses the briefest course. It would seem better to have printed the book by the loose leaf system so that the portions not needed by any particular school might be discarded.

I find many things to commend. The language is clear and to the point, and the instruction simple. The reference system to other books is very complete and capable of being very useful in schools supplied with libraries. The questions at the end of most of the studies are excellent in intent, but often are much too comprehensive for pupils of high school age. Teachers will be very glad to have a manual which incorporates such a useful set of experiments for the studies in physiology.

Part III on ecology is too much crowded and is hardly up to the standard of the rest of the book. Teachers of botany would gladly have seen a considerable reduction in Part II and a corresponding increase in this portion of the work. There is no doubt that much more might be done in this line than most schools are now doing and that it would greatly improve the course in botany for high school pupils.

While I find much to commend in the book, I cannot refrain from pointing out certain faults from the point of view of a *high school teacher*. The most serious fault perhaps has already been mentioned—the overloading of the book with many brief studies that not one high school teacher in twenty will ever think of using. When it is remembered that many teachers are not specialists in the subject, one cannot but feel that the authors should have been more "selective" in their part of the work. A second fault is in adding a large amount of instruction for the teacher—about forty-five pages being devoted to this. It must be remembered that the cost of school books is a serious item to many parents. The extra studies and these instructions add no inconsiderable amount to the cost of the book, and there is no doubt that this feature alone greatly lessens the usefulness of the book for high schools.

Two other points I wish to make concerning the presentation of the studies. Much of the fine type matter in the studies could well have been omitted—especially those in the type studies. These represent a class of work with the microscope that cannot profitably be done with high school classes. We ought to spend less time with the micro-

scope rather than more time. Our pupils are boys and girls—mostly girls—about 15 to 16 years of age, who know almost nothing of nature. What a pity to keep their eyes glued to microscopes in a laboratory, and how absurd to expect girls who cannot handle a jack knife to sharpen a lead pencil, to use a razor constantly in making sections fit for use in microscopic work. The authors are constantly throughout the book expecting too much of the pupils, and the teacher using the book would have to be alert to avoid the temptation to do too much.

The second suggestion is in regard to the questions. The authors do not seem to have followed any consistent plan. Pupils of high school age are entirely inexperienced in the use of laboratory outlines. They need very explicit and definite directions and questions. If the authors would enlarge the brief sets of questions at the end of the studies, and simplify them, they would make the studies much more practicable for high school use.

I do not wish to be understood as criticising the book for use in small colleges and some private schools. There is no doubt that the book is admirable for such a use. It is to be hoped that the authors may be prevailed upon to prepare a distinctively high school manual of botany, for we are sadly in need of such a book. It is a mistake to try to adapt a book to both older and younger pupils, but it is one that is often made, especially by biologists. W. W.

Plant Physiology and Ecology, by F. E. Clements. 8 vo, Pp. xv + 335, Illustrations 125. New York, Henry Holt & Co., 1907.

Teachers and students of botany will be interested in Clements's Plant Physiology and Ecology, which has just appeared. The book is written from the same standpoint as the author's Research Methods in Ecology, and may be regarded as a college text-book covering the same ground as the earlier book. It is stated in the preface that this work is intended for use in the second year of botany, with college and university classes, and that the manuscript for it has been so used with good success for the last two years in the University of Nebraska.

In the earlier chapters is given a discussion of various factors of the environment which influence the plant and of the possible responses brought about by changes in these factors. The later chapters discuss the plant formation and its behavior in connection with environmental factors, being thus a treatise on plant distribution. "Ecology and physiology are merged" in this book, and it will be seen that the discussion of the influences of environment upon the individual plant occupies a portion of the realm usually included in the latter science. In this discussion the author presents some of the elementary principles of stimulus and response, selecting those features for emphasis which are to be applied in the later chapters on distribution. Physiologists will probably take exception to the relative importance ascribed to the different kinds of response, and to many of the statements made in the discussion, but such critics are to remember that the book is primarily a text on distribution, despite the important position given in the title to the term physiology. The consideration of responses as

adjustments or adaptations makes it almost inevitable that a haze of teleology should hang over the presentation. Such statements as that the calcium consumed in chemosynthesis "is apparently for the purpose of neutralizing oxalic acid," etc., are particularly to be regretted in a text of ecology, a subject which needs emancipation from the teleological incubus perhaps more than any other of the subdivisions of botany.

The style of the work is exceedingly esoteric, and it will occur to many readers that the treatment is made unnecessarily forbidding by the preponderance of unfamiliar Greek technical terms recently put together by the author. Perhaps some may lose faith in the virtue of calling a spade a spade if they must perforce dub a familiar soil tube a *geotome*. The old *nyctitropic* has been reformed to the new *nyctotropic*. The illustrations are mainly excellent and well printed, but we meet here again a number of the old veterans, like Sachs's mustard seedlings, sanded and unsanded. In manufacture the book leaves nothing to be desired. It will take its place and be often referred to in all places where ecology is taught. B. E. LIVINGSTON.

AN ACADEMY OF SCIENCES FOR ILLINOIS.

A meeting has been called by Mr. A. R. Crook, curator of the state museum of natural history, of the men of science of Illinois, for Saturday, December 7, at 10 o'clock A. M., in the Senate chamber at Springfield, for the purpose of forming a state academy of sciences. A general invitation is extended to all persons interested in science to be present at the meeting.

The Southwestern Section of the American Mathematical Society will hold a meeting at St. Louis, November 29 and 30. The meeting of November 29 will be held in conjunction with the mathematical section of the Central Association at the McKinley High School. The Saturday meetings will be held in the building of the Washington University.

Do not forget the meeting of the Central Association of Science and Mathematics Teachers in St. Louis, November 29 and 30. If you have not received a program send for one at once to Mr. S. A. Douglass, Central High School, St. Louis, Mo.

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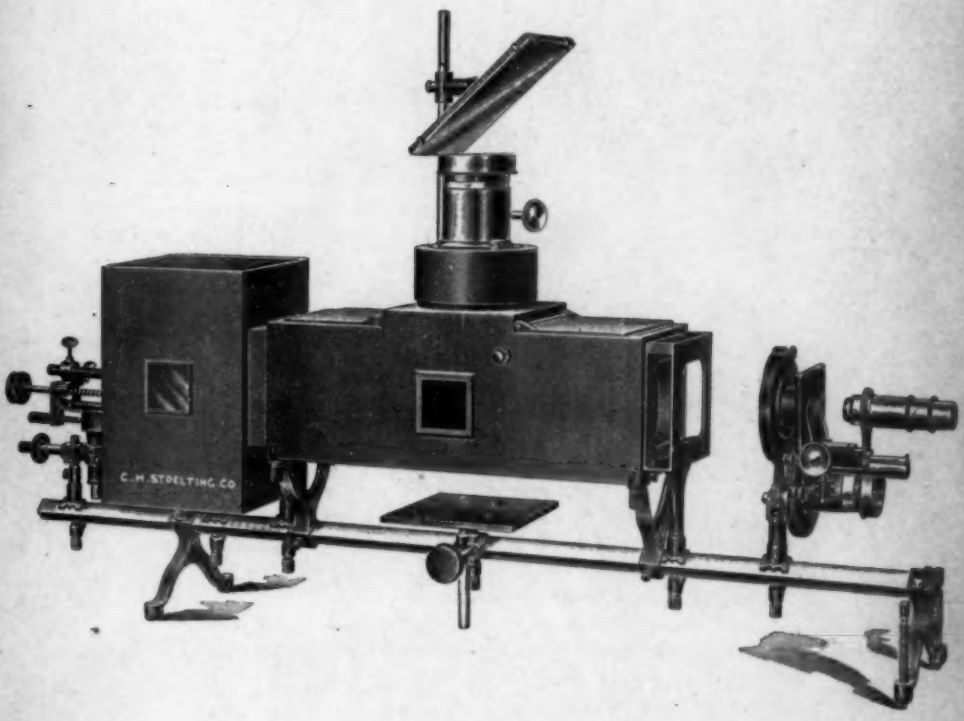

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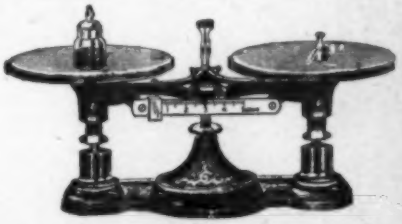

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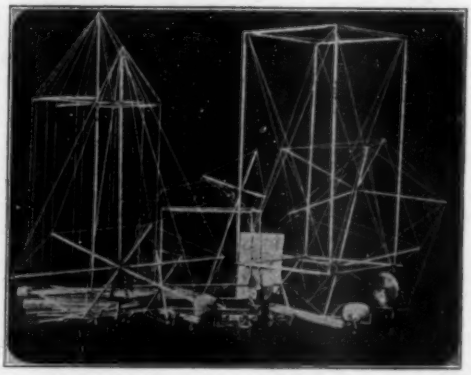

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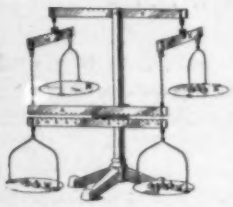

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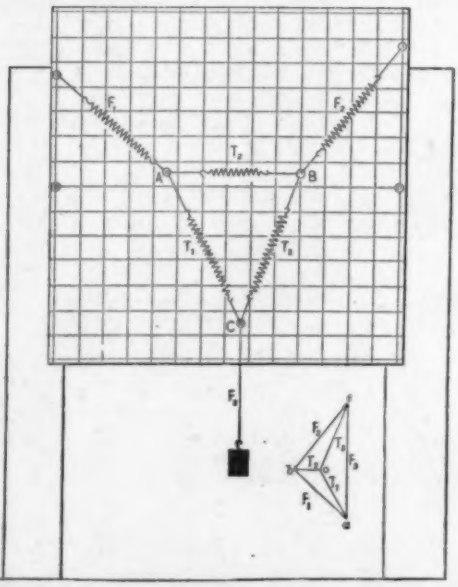

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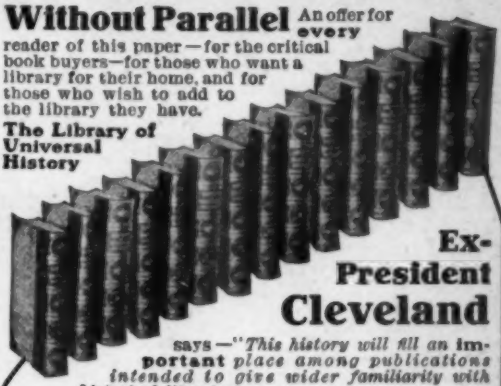

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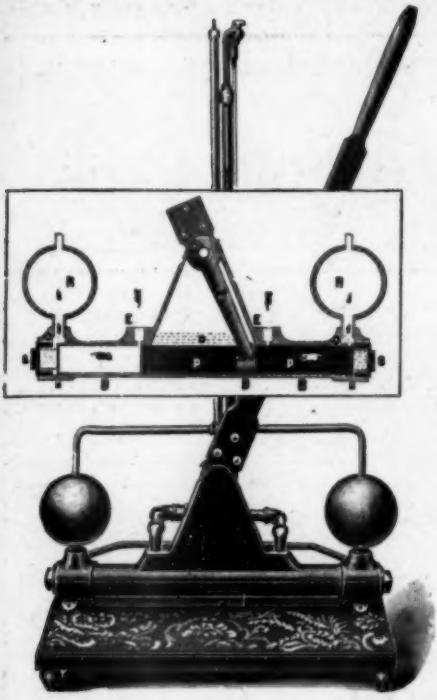

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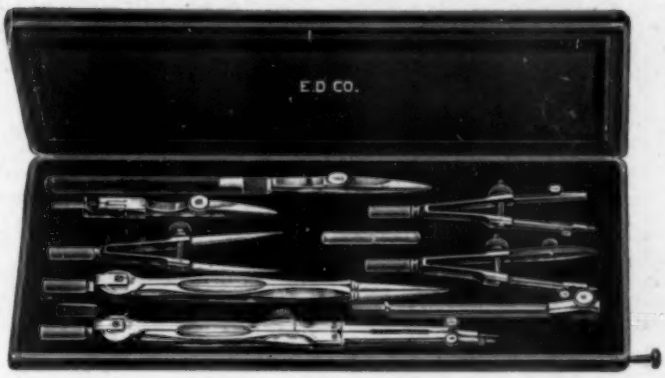

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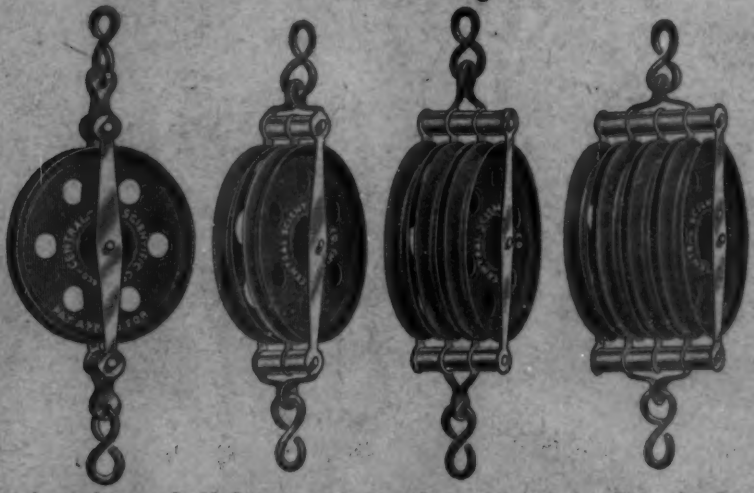

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